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ON CONVEX p-VALENT FUNCTIONS MAPPED ONTO THE NEPHROID DOMAIN

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ABSTRACT. p-valent functions serve as natural generalizations of univalent functions and offer a broad platform for studying geometric and functional properties within complex analysis. While coefficient estimation is a core problem in geometric function theory (GFT), the specific bounds and functional determinants for convex p-valent functions associated with the nephroid domain remain under-explored. This research addresses this gap by investigating a new subclass of functions characterized by subordination to a kidney-shaped region, which is motivated by the need to extend existing univalent results to broader p-valent classes. Using the theory of subordination and Taylor–Maclaurin series expansions, the methodology involves comparing the structural coefficients of convex p-valent functions against the nephroid-type mapping $P(\xi) = 1+\xi-\frac{\xi^3}{3}$. The author establishes estimates for the initial coefficients $|a_{1+p}|$ and $|a_{2+p}|$, and derives a generalized coefficient bound for $|a_{n+p}|$. These findings are verified for consistency by reducing the results to the specific univalent case where $p = 1$.

1. Introduction and Preliminaries

In the convenient and effective study of complex functions, their distinctive forms, and their applications, significant explorations have been carried out within the field of Complex Analysis, which remains a fundamental structure in Mathematics. Within this framework, the theory of analytic and multivalent functions remains a vibrant field of research, especially in Geometric Function Theory, GFT. p-valent functions, which are analytic in the open unit disk $E = \{\xi \in \mathbb{C} : |\xi| < 1\}$ and assume each value exactly p times, serve as natural generalizations of univalent (1-valent) functions and offer a broad platform for studying function theoretic and geometric properties ([3],[7]).

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The subclasses of these functions that are convex in the unit disk is a profuse one. Convexity is an important element in regularity geometry, and convex p-valent functions have long been studied in terms of coefficient bounds, distortion theorems, and mapping ([36],[34]). However, the estimation of initial coefficients remains a core problem, as these coefficients encode important geometric information about the function and its image.

The present paper proposes and discusses an interesting type of convex p-valent functions which identify with the domains of the nephroid domains. The primary objective is twofold: to obtain the generalization for the coefficients of functions in this class and to verify the findings using the specific univalent case. The findings presented here generalize results for convex functions in the univalent case, extend earlier investigations into p-valent subclasses, and contribute to ongoing research on functional determinants associated with geometrically constrained image domains ([33],[39]).

The interaction of geometry and analysis is possibly the most fascinating aspect of complex function theory. GFT is the branch of complex analysis that deals with the study of geometric properties of analytic functions. These functions are pivotal in the analysis of practical problems such as image processing and signal processing among others. The study of these geometric properties of analytic functions is deeply rooted in the estimation of Taylor-Maclaurin coefficients. The foundational work of Bieberbach in 1916 [1] regarding the coefficient bounds of univalent functions provided the initial impetus for this field. This was furthered by Fekete and Szegő [5] in 1933, who introduced the functional $|a_3 - \mu a_2^2|$, now a standard measure for the non-linearity of mappings. For multivalent functions, Goodman (1946) [6] extended these foundational estimates to the p-valent class, establishing a framework that remains central to modern research [12]-[30].

A significant area of development in recent years involves the study of functions subordinate to specific geometric domains. While circular and crescent-shaped regions have been extensively documented [32, 35], recent interest has shifted toward domains with characteristic symmetries and cusps. The nephroid domain, a kidney-shaped region, was first formally introduced into the context of starlike and convex functions by Wani and Swaminathan in 2020 [37]. They established the analytic mapping $p(\xi) = 1+\xi-\frac{\xi^3}{3}$ as the generating function for this domain, exploring its radius problems and structural properties.

Building upon this in [4], Fagbemiro et al. explored bi-univalent problems for generalized classes involving Q-integral operators specifically associated with the nephroid domain. Furthermore, the study of convex functions remains a prolific subfield due to their regularity in geometry. Classic results by Pommerenke [34] and Singh and Singh [36] laid the groundwork for coefficient bounds in multivalent convex classes. Recent investigations have continued to refine these bounds by applying them to symmetric and constrained domains, such as the sine function domain [2] and shell-like curves [22]. This research extends these cumulative efforts by specifically addressing the convex p-valent subclass within the nephroid region, filling a notable gap in the existing literature regarding higher-order coefficient generalizations.

Suppose that Υ represent the class of all analytic p-valent functions $f_p(\xi)$, having the series notation

$$f_p(\xi) = \xi^p + \sum_{k=1}^{\infty} a_{k+p}\xi^{k+p}. \tag{1.1}$$

For recent work on p-valent functions, refer to [8]-[11].
Specifically, when $p = 1$, we can denote the class of all Analytic Functions that are univalent in the open disk $E = \{\xi \in \mathbb{C} : |\xi| < 1\}$ by S, with the functions of the form

$$f(\xi) = \xi + \sum_{k=2}^{\infty} a_k\xi^k \tag{1.2}$$

and normalized by $f'(0) - 1 = 0 = f(0)$.
For two functions $g, h \in \Upsilon$, we say the function g is subordinate to the function h (written as $g \prec h$) if there exists an analytic function ω with the property

$$|\omega(\xi)| \leq 1 \quad and \quad \omega(\xi) = 0$$

such that

$$g(\xi) = h(w(\xi)) \quad (\xi \in E)$$

. Let $\mathfrak{P}$ be the class of analytic functions with positive real part, normalized by

$$p(0) = 1 \quad with \quad \Re(p(\xi)) > 0 \quad (\xi \in E)$$

and have the following form:

$$p(\xi) = 1 + \sum_{n=1}^{\infty} c_n\xi^n \quad (\xi \in E). \tag{1.3}$$

Also, let Θ represent the family of convex p-valent functions such that for each function $f_p \in \Theta$,

$$1 + \frac{\xi f_p''(\xi)}{f_p'(\xi)} \prec p(\xi),$$

where $p(\xi) \in \mathfrak{P}$.
It should be noted that there are numerous functions that form subclasses of $\mathfrak{P}$, including but not limited to:

- (see [32], [38]) The circular domain centered on $\frac{1-AB}{1-B^2}$ and radius $\frac{A-B}{1-B^2}$ of the form
$$p(\xi) = \frac{1 + A\xi}{1 + B\xi}; \quad -1 \leq B < A \leq 1.$$
- (see [35]) The crescent-shaped region of the form
$$p(\xi) = \xi + \sqrt{1 + \xi^2}.$$
- (see [2]) The eight-shaped region of the form
$$p(\xi) = 1 + sin\xi.$$

- (see [37]) The nephroid domain of the form

$$p(\xi) = 1 + \xi - \frac{\xi^3}{3}.$$

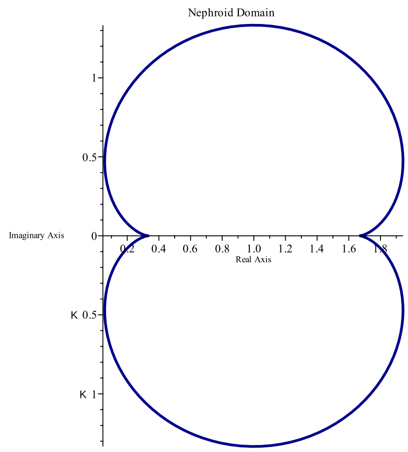

FIGURE 1. Nephroid domain generated by the mapping $p(\xi) = 1 + \xi - \frac{\xi^3}{3}$ for $|\xi| = 1$.

Figure 1 illustrates the nephroid domain obtained as the image of the unit circle under the analytic mapping

$$p(\xi) = 1 + \xi - \frac{\xi^3}{3}.$$

The boundary of the domain is traced by letting $\xi = e^{i\theta},\ 0 \leq \theta \leq 2\pi$, which produces a smooth, symmetric nephroid curve. This mapping preserves univalence on the unit circle and generates a domain with characteristic cusps and rotational symmetry, making it suitable for studying coefficient bounds of convex p-valent functions subordinate to nephroid-type regions. The geometry of the image highlights how higher-order terms influence boundary curvature and distortion, which is central to the coefficient estimation results established in the preceding theorems.

Although the functions above have been used widely in literature, this work focuses on estimation of the coefficients of convex p-valent functions mapped on the nephroid kind.

Let N be the class of analytic p-valent functions f_p of the form (1.1) associated with the nephroid domain and $f_p \prec P(\xi)$, with $P(\xi)$ given by

$$P(\xi) = 1 + \xi - \frac{\xi^3}{3} \tag{1.4}$$

denoting the nephroid domain.

Definition 1:
Let C_N be the class of convex p-valent functions associated with the nephroid domain, satisfying the geometric condition:

$$\frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} \prec P(\xi), \tag{1.5}$$

.

The following lemmas are essential to prove our results.

Lemma 1.1. *([34]): Let $p \in \mathfrak{P}$. Then, the following inequalities hold*

$$|c_k| \leq 2 \quad for \quad k \geq 1, \tag{1.6}$$

$$|c_{n+k} - \mu c_n c_k| < 2 \quad for \quad 0 \leq \mu \leq 1, \tag{1.7}$$

$$|c_m c_k - c_k c_1| \leq 4 \quad for \quad m + k = k + l, \tag{1.8}$$

$$|c_{k+2k} - \mu c_k c_k^2| \leq 2(1 + 2\mu) \quad for \quad \mu \in \mathbb{R}, \tag{1.9}$$

$$\left| c_2 - \frac{c_1^2}{2} \right| \leq 2 - \frac{|c_1^2|}{2}, \tag{1.10}$$

Lemma 1.2. *([31]): Let $p \in \mathfrak{P}$, then for complex number η, we have*

$$|c_2 - \eta c_1^2| < 2max\{1, |2\eta - 1|\}. \tag{1.11}$$

2. Main Results

Theorem 2.1. *Let $f_p \in C_N$ and of the form 1.1. Then*

$$|a_{1+p}| \leq \frac{p^2}{(1 + p)};$$

$$|a_{2+p}| \leq \frac{p^2}{2(2 + p)} max\{1, p\}.$$

Proof. Since $f_p \in C_N$, then

$$\frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} \prec P(\xi),$$

where $P(\xi)$ is given by 1.4.
There exists an analytic function $\omega(\xi) = b_1\xi + b_2\xi^2 + b_3\xi^3 + ...$ with the property

$$|\omega(\xi)| \leq 1 \quad and \quad \omega(\xi) = 0, \ for \ \xi \in E$$

such that

$$p(\xi) = \frac{1 + \omega(\xi)}{1 + \omega(\xi)} = 1 + c_1\xi + c_2\xi^2 + c_3\xi^3 + ... \tag{2.1}$$

Making $\omega(\xi)$ subject of formula in 2.1 gives

$$\omega(\xi) = \frac{p(\xi)-1}{p(\xi)+1} = b_1\xi + b_2\xi^2 + b_3\xi^3 + ...$$

where

$$b_1 = \frac{1}{2}c_1;\ b_2 = \frac{1}{4}(2c_2 - c_1^2);\ b_3 = \frac{1}{8}(4c_3 - 4c_1c_2 + c_1^3)$$

and so on. Now,

$$P(\omega(\xi)) = 1 + \omega(\xi) - \frac{(\omega(\xi))^3}{3}$$

Some basic computations break this function into series form as

$$P(\omega(\xi)) = 1 + b_1\xi + b_2\xi^2 + \left(b_3 - \frac{b_1^3}{3}\right)\xi^3 + ... \tag{2.2}$$

Also, for $f_p \in C_N$ and of the form 1.1, it is easy to see that

$$\frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} = 1 + \frac{1+p}{p^2}a_{1+p}\xi + \frac{1}{p^2}\left[2(2+p)a_{2+p} - \frac{(1+p)^2}{p}(a_{1+p})^2\right]\xi^2 + \frac{1}{p^2}\left[3(3+p)a_{3+p} - \frac{3(1+p)(2+p)}{p}a_{1+p}a_{2+p} + \frac{(1+p)^3}{p^2}(a_{1+p})^3\right]\xi^3 + ... \tag{2.3}$$

Comparing the coefficients from 2.2 and 2.3, we have that

$$a_{1+p} = \frac{c_1p^2}{2(1+p)};$$

$$a_{2+p} = \frac{p^2}{4(2+p)}\left(c_2 - \frac{1}{2}(1-p)c_1^2\right);$$

Let

$$\eta = \frac{1}{2}(1-p)$$

. Then, using Lemmas 1.1 and 1.2, the required results are obtained. □

Theorem 2.2. *Let $f_p \in C_N$ and of the form 1.1. Then for $n = 2, 3, 4, ...$,*

$$|a_{n+p}|^2 \leq \frac{1}{9n^2(n+p)^2}\left[((9p^4 + 9(p-n+1)(p+n-1)^3|a_{p+n-1}|^2 + \sum_{k=1}^{n-2}[(9p^2 - 9k^2)(k+p)^2]|a_{k+p}|^2\right]$$

Proof. Since $f_p \in C_N$, then

$$\frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} \prec P(\xi) \implies \frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} \prec 1 + \xi - \frac{\xi^3}{3}$$

It follows that there exists an analytic function ω with the property

$$|\omega(\xi)| \leq 1 \quad and \quad \omega(\xi) = 0$$

such that

$$\frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} \prec 1+\xi-\frac{\xi^3}{3} \implies \frac{(\xi f_p'(\xi))'}{pf_p'(\xi)} = 1+\omega(\xi)-\frac{(\omega(\xi))^3}{3}$$

From the process of cross multiplication and combination of like terms, we have

$$3((\xi f_p'(\xi))' - pf_p'(\xi)) = 3\omega(\xi)pf_p'(\xi) - (\omega(\xi))^3 pf_p'(\xi) \tag{2.4}$$

Of course, from 1.1

$$f_p(\xi) = \xi^p + \sum_{k=1}^{\infty} a_{k+p}\xi^{k+p} \implies f_p'(\xi) = p\xi^{p-1} + \sum_{k=1}^{\infty}(k+p)a_{k+p}\xi^{k+p-1}$$

$$\implies pf_p'(\xi) = p^2\xi^{p-1} + \sum_{k=1}^{\infty} p(k+p)a_{k+p}\xi^{k+p-1}; and$$

$$\xi f_p'(\xi) = p\xi^p + \sum_{k=1}^{\infty}(k+p)a_{k+p}\xi^{k+p}$$

$$\implies (\xi f_p'(\xi))' = p^2\xi^{p-1} + \sum_{k=1}^{\infty}(k+p)^2 a_{k+p}\xi^{k+p-1}$$

Now 2.4 becomes

$$\sum_{k=1}^{\infty} 3k(k+p)a_{k+p}\xi^{k+p} = 3\omega(\xi)[p^2\xi^p + \sum_{k=1}^{\infty} p(k+p)a_{k+p}\xi^{k+p}] - (\omega(\xi))^3[p^2\xi^p + \sum_{k=1}^{\infty} p(k+p)a_{k+p}\xi^{k+p}]$$

Splitting the summands, we then have that

$$\sum_{k=1}^{n} 3k(k+p)a_{k+p}\xi^{k+p} + \sum_{k=n+1}^{\infty} 3k(k+p)a_{k+p}\xi^{k+p} = 3(\omega(\xi))[p^2\xi^p + \sum_{k=1}^{n-1} p(k+p)a_{k+p}\xi^{k+p} + \sum_{k=n}^{\infty} p(k+p)a_{k+p}\xi^{k+p}] - (\omega(\xi))^3[p^2\xi^p + \sum_{k=1}^{n-2} p(k+p)a_{k+p}\xi^{k+p} + \sum_{k=n-1}^{\infty} p(k+p)a_{k+p}\xi^{k+p}] \tag{2.5}$$

Let

$$\sum_{k=n+1}^{\infty} b_{k+p}\xi^{k+p} = \sum_{k=n+1}^{\infty} 3k(k+p)a_{k+p}\xi^{k+p} - 3(\omega(\xi))\sum_{k=n}^{\infty} p(k+p)a_{k+p}\xi^{k+p} + (\omega(\xi))^3 \sum_{k=n-1}^{\infty} p(k+p)a_{k+p}\xi^{k+p}$$

From 2.5, it follows that

$$\sum_{k=1}^{n} 3k(k+p)a_{k+p}\xi^{k+p} + \sum_{k=n+1}^{\infty} b_{k+p}\xi^{k+p} = (3\omega(\xi) - (\omega(\xi))^3)p^2\xi^p \\ + 3p(p+n-1)\omega(\xi)a_{p+n-1}\xi^{p+n-1} \\ + \sum_{k=1}^{n-2}[3\omega(\xi) - (\omega(\xi))^3]p(k+p)a_{k+p}\xi^{k+p}$$

So,

$$\left|\sum_{k=1}^{n} 3k(k+p)a_{k+p}\xi^{k+p} + \sum_{k=n+1}^{\infty} b_{k+p}\xi^{k+p}\right|^2 = \Big|(3\omega(\xi) - (\omega(\xi))^3)p^2\xi^p \\ + 3p(p+n-1)\omega(\xi)a_{p+n-1}\xi^{p+n-1} \\ + \sum_{k=1}^{n-2}[3\omega(\xi) - (\omega(\xi))^3]p(k+p)a_{k+p}\xi^{k+p}\Big|^2 \tag{2.6}$$

Since $|\omega(\xi)| \leq 1$, set

$$d_{k+p} = \begin{cases} 3k(k+p)a_{k+p} & \text{if } 1 \leq k \leq n \\ b_{k+p} & \text{if } k \geq n+1, \end{cases}$$

then 2.6 becomes

$$\left|\sum_{k=1}^{\infty} d_{k+p}\xi^{k+p}\right|^2 < \Big|3p^2\xi^p + 3p(p+n-1)a_{p+n-1}\xi^{p+n-1} \\ + \sum_{k=1}^{n-2}[3-(\omega(\xi))^2]p(k+p)a_{k+p}\xi^{k+p}\Big|^2$$

Using $\frac{1}{2\pi}\int_0^{2\pi} |\sum_{k=1}^{\infty} d_{k+p}(re^{i\theta})^{k+p}|^2\, d\theta = \sum_{k=1}^{\infty} |d_{k+p}|^2 r^{2(k+p)}$(see [7]) and then integrating on $\xi = re^{i\theta}, where\ |e^{i\theta}| = 1,\ 0 < r < 1,\ and\ 0 \leq \theta < 2\pi$, we then have

$$\sum_{k=1}^{\infty}|d_{k+p}|^2r^{2(k+p)} < \frac{1}{2\pi}\int_0^{2\pi}\Bigg|3p^2(re^{i\theta})^p + 3p(p+n-1)a_{p+n-1}(re^{i\theta})^{p+n-1}$$

$$+\sum_{k=1}^{n-2}[3-(\omega(re^{i\theta}))^2]p(k+p)a_{k+p}(re^{i\theta})^{k+p}\Bigg|^2 d\theta$$

$$< \frac{1}{2\pi}\int_0^{2\pi}\Bigg[\Bigg(3p^2r^pe^{ip\theta} + 3p(p+n-1)a_{p+n-1}r^{p+n-1}e^{i(p+n-1)\theta}$$

$$+\sum_{k=1}^{n-2}[3-(\omega(re^{i\theta}))^2]p(k+p)a_{k+p}r^{k+p}e^{i(k+p)\theta}\Bigg)\times$$

$$\Bigg(3p^2r^pe^{-ip\theta} + 3p(p+n-1)a_{p+n-1}r^{p+n-1}e^{-i(p+n-1)\theta}$$

$$+\sum_{l=1}^{n-2}[3-(\omega(re^{i\theta}))^2]p(l+p)a_{l+p}r^{l+p}e^{-i(l+p)\theta}\Bigg)\Bigg]d\theta$$

Since the integral of product with $k \neq l$ gives 0, it follows that

$$\sum_{k=1}^{n}|3k(k+p)a_{k+p}|^2r^{2(k+p)} + \sum_{k=n+1}^{\infty}|b_{k+p}|^2r^{2(k+p)} < (3p^2r^p)^2 + |3p(p+n-1)|^2|a_{p+n-1}|^2r^{2(p+n-1)}$$

$$+\sum_{k=1}^{n-2}|3-(\omega(\xi))^2|^2|p(k+p)|^2|a_{k+p}|^2r^{2(k+p)}$$

By triangle inequality, since $|\omega(\xi)| \leq 1$, we have

$$\sum_{k=1}^{n}|3k(k+p)a_{k+p}|^2r^{2(k+p)} + \sum_{k=n+1}^{\infty}|b_{k+p}|^2r^{2(k+p)} \leq 9p^4r^{2p} + 9p^2(p+n-1)^2|a_{p+n-1}|^2r^{2(p+n-1)}$$

$$+\sum_{k=1}^{n-2}9p^2(k+p)^2|a_{k+p}|^2r^{2(k+p)}$$

which then reduces to

$$9\sum_{k=1}^{n}k^2(k+p)^2|a_{k+p}|^2r^{2(k+p)} \leq 9p^4r^{2p} + 9p^2(p+n-1)^2|a_{p+n-1}|^2r^{2(p+n-1)}$$

$$+\sum_{k=1}^{n-2}9p^2(k+p)^2|a_{k+p}|^2r^{2(k+p)}$$

Setting $r \to 1$ and then unifying the sums give

$$9n^2(n+p)^2|a_{n+p}|^2 + 9(n-1)^2(p+n-1)^2|a_{p+n-1}|^2 + 9\sum_{k=1}^{n-2} k^2(k+p)^2|a_{k+p}|^2 r^{2(k+p)}$$
$$\leq 9p^4 + 9p^2(p+n-1)^2|a_{p+n-1}|^2 + \sum_{k=1}^{n-2} 9p^2(k+p)^2|a_{k+p}|^2$$

From here, the required result is easily obtained. □

It will be interesting to note that for a specific value of p, the following corollary can be deduced.

Corollary 2.3. *Let f be a convex univalent function (i.e. $p = 1$) and of the form (1.2). Then for $n = 2, 3, 4, \ldots$,*

$$|a_{n+1}|^2 \leq \frac{1}{9n^2(n+1)^2}\left[((9 - 9n^4|a_n|^2 + \sum_{k=1}^{n-2}[(9-9k^2)(k+1)^2]|a_{k+1}|^2\right]$$

3. Conclusion

In this study, precise estimates for the initial Taylor–Maclaurin coefficients $|a_{1+p}|$ and $|a_{2+p}|$ for the class of convex p-valent functions subordinate to the nephroid domain were established. Furthermore, a generalized coefficient bound for $|a_{n+p}|$ was derived, providing a comprehensive characterization of the function's structural properties and growth.

These findings have practical applications in the broader field of complex analysis, as the estimated coefficients encode vital geometric information regarding the curvature and distortion of the function's image. Additionally, the established bounds and functional determinants are instrumental for calculating image enhancement problems and evaluating the behavior of linear operators within geometrically constrained domains.

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