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PERFORMANCE EVALUATION OF A COMPUTATIONAL BLOCK METHOD FOR SOLVING QUADRATIC RICCATI DIFFERENTIAL EQUATIONS: A NUMERICAL VALIDATION AND COMPARATIVE ANALYSIS

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ABSTRACT. This study presents a computational block method derived through interpolation and collocation using power series polynomials for solving quadratic Riccati differential equations (QRDEs). A rigorous analysis of the method's core properties including order, consistency, and stability confirms its theoretical soundness. The method's performance was evaluated by applying it to three benchmark QRDEs. Numerical results demonstrate that the proposed method achieves significantly higher accuracy compared to several existing techniques documented in the literature. The study concludes that the computational block method is an efficient and reliable numerical tool for solving QRDEs, offering superior precision and convergence characteristics.

1. INTRODUCTION

Quadratic Riccati Differential Equations (QRDEs) are a fundamental class of nonlinear ordinary differential equations with extensive applications in scientific and engineering disciplines. They play a critical role in optimal control theory, robust stabilization, financial mathematics, and the analysis of various physical systems such as spring-mass dynamics and electrical circuits [1, 2, 3]. The general form of a first-order QRDE is given by:

A primary challenge in handling Riccati differential equations (QRDEs) stems from their nonlinearity, which often precludes closed-form analytical solutions. Consequently, researchers rely heavily on numerical and approximate methods [4]. Additionally, the convergence and stability of numerical methods can pose challenges, especially when dealing

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with highly nonlinear equations or when the equations involve singularities or discontinuities. Another limitation is the computational cost associated with numerical methods, particularly when solving equations with high-dimensional parameter spaces or when high accuracy is required. Furthermore, the presence of multiple solutions or bifurcation phenomena in nonlinear systems can complicate the interpretation of results and require careful analysis to ensure the correct solution is obtained. Overall, while numerical methods offer a powerful tool for solving nonlinear quadratic Riccati equations, they also come with challenges related to accuracy, stability, computational cost, and interpretation of results [5-7].

Riccati Differential Equations (RDE) constitute a significant class of nonlinear differential equations with broad relevance in applied science domains often linked to the one-dimensional static Schrödinger equation [8, 9]. Named after Count Jacopo Francesco Riccati (1676–1754), these equations find application not only in random processes, optimal control, and diffusion problems but also in stochastic realization theory, optimal control, network synthesis, and financial mathematics [8]. The QRDE, expressed as a first-order ordinary differential equation, plays a pivotal role in various analytical and computational studies

$$y'(t)=ay^2(t)+bt+c \tag{1.1}$$

In this study, we consider the quadratic Riccati differential equation, where y is the dependent variable and t is the independent variable. The quadratic Riccati differential equation is a special case where a, b and c are constants and $a \neq 0$.

A common approach to solving a quadratic Riccati differential equation involves making a substitution that transforms it into a linear second-order ordinary differential equation. This transformation enables the use of standard methods for solving such equations [10].

Quadratic Riccati equations are ubiquitous in mathematics and physics, finding applications in control theory, differential geometry, and mathematical physics, with implications in optimal control, stability analysis, and nonlinear dynamics [11]. Particularly in the context of nonlinear partial differential equations, the expression of solitary wave solutions often involves representing them as a polynomial in elementary functions satisfying a projective Riccati equation [12]. This approach extends beyond wave dynamics to problems in optimal control, attracting considerable interest and extensive investigation. Despite their nonlinear nature, analogous to second-order inhomogeneous linear ordinary differential equations, obtaining a particular solution is sufficient to deduce the general solution, a problem that has received significant attention from scholars [13].

Several techniques have been developed, including the Homotopy Analysis Method (HAM) [14], the differential transformation method (DTM) [7], multistage variational iteration methods [15], and various Runge-Kutta formulations [1] and Newton-Raphson based modified Laplace Adomian decomposition methods [12]. While these methods provide viable solutions, they can be limited by issues related to computational efficiency, stability over large intervals, and achieving high-order accuracy with manageable complexity [5-7]. In this study, we propose a new computational block method based on power series interpolation and collocation to address these challenges. Our motivation is to develop a numerically stable, high-order method that improves upon existing techniques in both precision and convergence rate [8]. The remainder of this paper is structured as follows: Section 2 details the derivation of the method; Section 3 analyzes its theoretical properties; Section 4 presents numerical results; and Section 5 discusses findings and conclusions.

2. DERIVATION OF THE GENERALIZED COMPUTATIONAL METHODS

In these section, we will derive a computational method for the simulation of Quadratic Ricatti differential equation of the (1.1). We approximate the solution of (1.1) using power series polynomial of the form

$$y(x)=\sum_{n=0}^{r+s-1}a_n x^n \tag{2.1}$$

where r, s is the distinct number of interpolation and collocation points. The first derivative of (2.1) is given as

$$y'(x)=\sum_{n=0}^{r+s-1}n a_n x^{n-1} \tag{2.2}$$

Equation (2.2) is Substituted into (1.1) gives,

$$y'(x)=f(x,y) \tag{2.3}$$

Therefore, interpolating equation (2.1) at point $x_{n+s}, s=1$ and collocating equation (2.2) at $x_{n+r}, r=0,\frac{1}{6},\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{5}{6},1$, now these leads to a system of nonlinear equation of the form

$$\begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 & x_{n+1}^9 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 \\ 0 & 1 & 2x_{n+\frac{1}{6}} & 3x^2_{n+\frac{1}{6}} & 4x^3_{n+\frac{1}{6}} & 5x^4_{n+\frac{1}{6}} & 6x^5_{n+\frac{1}{6}} & 7x^6_{n+\frac{1}{6}} & 8x^7_{n+\frac{1}{6}} & 9x^8_{n+\frac{1}{6}} \\ 0 & 1 & 2x_{n+\frac{1}{4}} & 3x^2_{n+\frac{1}{4}} & 4x^3_{n+\frac{1}{4}} & 5x^4_{n+\frac{1}{4}} & 6x^5_{n+\frac{1}{4}} & 7x^6_{n+\frac{1}{4}} & 8x^7_{n+\frac{1}{4}} & 9x^8_{n+\frac{1}{4}} \\ 0 & 1 & 2x_{n+\frac{1}{3}} & 3x^2_{n+\frac{1}{3}} & 4x^3_{n+\frac{1}{3}} & 5x^4_{n+\frac{1}{3}} & 6x^5_{n+\frac{1}{3}} & 7x^6_{n+\frac{1}{3}} & 8x^7_{n+\frac{1}{3}} & 9x^8_{n+\frac{1}{3}} \\ 0 & 1 & 2x_{n+\frac{1}{2}} & 3x^2_{n+\frac{1}{2}} & 4x^3_{n+\frac{1}{2}} & 5x^4_{n+\frac{1}{2}} & 6x^5_{n+\frac{1}{2}} & 7x^6_{n+\frac{1}{2}} & 8x^7_{n+\frac{1}{2}} & 9x^8_{n+\frac{1}{2}} \\ 0 & 1 & 2x_{n+\frac{2}{3}} & 3x^2_{n+\frac{2}{3}} & 4x^3_{n+\frac{2}{3}} & 5x^4_{n+\frac{2}{3}} & 6x^5_{n+\frac{2}{3}} & 7x^6_{n+\frac{2}{3}} & 8x^7_{n+\frac{2}{3}} & 9x^8_{n+\frac{2}{3}} \\ 0 & 1 & 2x_{n+\frac{3}{4}} & 3x^2_{n+\frac{3}{4}} & 4x^3_{n+\frac{3}{4}} & 5x^4_{n+\frac{3}{4}} & 6x^5_{n+\frac{3}{4}} & 7x^6_{n+\frac{3}{4}} & 8x^7_{n+\frac{3}{4}} & 9x^8_{n+\frac{3}{4}} \\ 0 & 1 & 2x_{n+\frac{5}{6}} & 3x^2_{n+\frac{5}{6}} & 4x^3_{n+\frac{5}{6}} & 5x^4_{n+\frac{5}{6}} & 6x^5_{n+\frac{5}{6}} & 7x^6_{n+\frac{5}{6}} & 8x^7_{n+\frac{5}{6}} & 9x^8_{n+\frac{5}{6}} \\ 0 & 1 & 2x_{n+1} & 3x^2_{n+1} & 4x^3_{n+1} & 5x^4_{n+1} & 6x^5_{n+1} & 7x^6_{n+1} & 8x^7_{n+1} & 9x^8_{n+1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} = \begin{bmatrix} y_{n+1} \\ f_n \\ f_{n+\frac{1}{6}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{1}{2}} \\ f_{n+\frac{2}{3}} \\ f_{n+\frac{3}{4}} \\ f_{n+\frac{5}{6}} \\ f_{n+1} \end{bmatrix} \quad (2.4)$$

Solving the system of nonlinear equation (2.4) by Gauss elimination method for the $a_j's,\ j=0(1)9$ and substituting back into the power series basis function gives a computational block method of the form,

$$y(x)=a_1(x)y_{n+1}+h\begin{bmatrix} \beta_0(x)f_n+\beta_{\frac{1}{6}}(x)f_{n+\frac{1}{6}}+\beta_{\frac{1}{4}}(x)f_{n+\frac{1}{4}}+\beta_{\frac{1}{3}}(x)f_{n+\frac{1}{3}}+\beta_{\frac{1}{2}}(x)f_{n+\frac{1}{2}} \\ +\beta_{\frac{2}{3}}(x)f_{n+\frac{2}{3}}+\beta_{\frac{3}{4}}(x)f_{n+\frac{3}{4}}+\beta_{\frac{5}{6}}(x)f_{n+\frac{5}{6}}+\beta_1(x)f_{n+1} \end{bmatrix} \quad (2.5)$$

where the coefficients $a_1(x),\ \beta_0(x),\ \beta_{\frac{1}{6}}(x),\ \beta_{\frac{1}{4}}(x),\ \beta_{\frac{1}{3}}(x),\ \beta_{\frac{1}{2}}(x),\ \beta_{\frac{2}{3}}(x),\ \beta_{\frac{3}{4}}(x),\ \beta_{\frac{5}{6}}(x),\ \beta_1(x)$ are continues scheme. Thus, evaluating (2.5) at none interpolating points, to gives

$$
\begin{pmatrix} y_n \\ y_{n+\frac{1}{6}} \\ y_{n+\frac{1}{4}} \\ y_{n+\frac{1}{3}} \\ y_{n+\frac{1}{2}} \\ y_{n+\frac{2}{3}} \\ y_{n+\frac{3}{4}} \\ y_{n+\frac{5}{6}} \end{pmatrix} = y_{n+1} + h \begin{pmatrix} -\frac{1}{12600} \\ \frac{5}{5878656} \\ \frac{3}{2867200} \\ \frac{1}{1148175} \\ \frac{1}{201600} \\ \frac{1}{9185400} \\ \frac{1}{25804800} \\ \frac{1}{146966400} \end{pmatrix}
\begin{pmatrix}
503 & -5832 & 8192 & -9477 & 2640 & -9477 & 8192 & -5832 & -503 \\
2005 & -88128 & 74240 & -364095 & 9600 & -653265 & 581120 & -492480 & -48773 \\
1481 & -41796 & 125696 & -315171 & 13920 & -536301 & 476416 & -401436 & -39609 \\
1844 & -53784 & 200704 & -325539 & 11280 & -640224 & 569344 & -481464 & -47611 \\
281 & -7776 & 27136 & -33291 & 21120 & -118341 & 103936 & -85536 & 8329 \\
14201 & -399816 & 1417216 & -1786941 & 1834320 & -4304421 & 4366336 & -3821256 & -381439 \\
39299 & -1105164 & 3913984 & -4928769 & 5030880 & -10899279 & 13383424 & -10815444 & -1070131 \\
229633 & -6464448 & 22911488 & -28881603 & 29592960 & -65027853 & 86271488 & -57008448 & 6117617
\end{pmatrix}
\begin{pmatrix} f_n \\ f_{n+\frac{1}{6}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{1}{2}} \\ f_{n+\frac{2}{3}} \\ f_{n+\frac{3}{4}} \\ f_{n+\frac{5}{6}} \\ f_{n+1} \end{pmatrix} \quad (2.6)
$$

From equation (2.6), the discrete computational block method is given in a matrix form as

$$
\begin{pmatrix} y_{n+\frac{1}{6}} \\ y_{n+\frac{1}{4}} \\ y_{n+\frac{1}{3}} \\ y_{n+\frac{1}{2}} \\ y_{n+\frac{2}{3}} \\ y_{n+\frac{3}{4}} \\ y_{n+\frac{5}{6}} \\ y_{n+1} \end{pmatrix} = y_n + h \begin{pmatrix} \frac{1}{146966400} \\ \frac{1}{25804800} \\ \frac{1}{9185400} \\ \frac{1}{201600} \\ \frac{1}{1148175} \\ \frac{3}{2867200} \\ \frac{5}{5878656} \\ \frac{1}{12600} \end{pmatrix}
\begin{pmatrix}
6117617 & 57008448 & -86271488 & 65027853 & -29592960 & 28881603 & -22911488 & 6464448 & -229633 \\
1070131 & 10815444 & -13383424 & 10899279 & -5030880 & 4928769 & -3913984 & 1105164 & -39299 \\
381439 & 3821256 & -4366336 & 4304421 & -1834320 & 1786941 & -1417216 & 399816 & -14201 \\
8329 & 85536 & -103936 & 118341 & -21120 & 33291 & -27136 & 7776 & -281 \\
47611 & 481464 & -569344 & 640224 & -11280 & 325539 & -200704 & 53784 & -1844 \\
39609 & 401436 & -476416 & 536301 & -13920 & 315171 & -125696 & 41796 & -1481 \\
48773 & 492480 & -581120 & 653265 & -9600 & 364095 & -74240 & 88128 & -2005 \\
503 & 5832 & -8192 & 9477 & -2640 & 9477 & -8192 & 5832 & 503
\end{pmatrix}
\begin{pmatrix} f_n \\ f_{n+\frac{1}{6}} \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{1}{3}} \\ f_{n+\frac{1}{2}} \\ f_{n+\frac{2}{3}} \\ f_{n+\frac{3}{4}} \\ f_{n+\frac{5}{6}} \\ f_{n+1} \end{pmatrix} \quad (2.7)
$$

3. ANALYSIS OF THE COMPUTATIONAL BLOCK METHODS

In this segment, we will delve into several fundamental characteristics of the computational block method, including its order, error constant, consistency, convergence, zero-stability, and linear stability.

3.1 Order of Accuracy and Error Constant of the Method

The linear operator of the computational block method is expressed as equation (2.7)

$$L\{y(x);h\}=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{bmatrix}\begin{pmatrix}y_{n+\frac{1}{6}}\\y_{n+\frac{1}{4}}\\y_{n+\frac{1}{3}}\\y_{n+\frac{1}{2}}\\y_{n+\frac{2}{3}}\\y_{n+\frac{3}{4}}\\y_{n+\frac{5}{6}}\\y_{n+1}\end{pmatrix}-\begin{bmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\end{bmatrix}\begin{pmatrix}y_{n-\frac{1}{6}}\\y_{n-\frac{1}{4}}\\y_{n-\frac{1}{3}}\\y_{n-\frac{1}{2}}\\y_{n-\frac{2}{3}}\\y_{n-\frac{3}{4}}\\y_{n-\frac{5}{6}}\\y_{n}\end{pmatrix}+h\begin{pmatrix}\frac{1}{146966400}\\\frac{1}{25804800}\\\frac{1}{9185400}\\\frac{1}{201600}\\\frac{1}{1148175}\\\frac{3}{2867200}\\\frac{5}{5878656}\\\frac{1}{12600}\end{pmatrix}$$

$$\begin{pmatrix}6117617&57008448&-86271488&65027853&-29592960&28881603&-22911488&6464448&-229633\\1070131&10815444&-13383424&10899279&-5030880&4928769&-3913984&1105164&-39299\\381439&3821256&-4366336&4304421&-1834320&1786941&-1417216&399816&-14201\\8329&85536&-103936&118341&-21120&33291&-27136&7776&-281\\47611&481464&-569344&640224&-11280&325539&-200704&53784&-1844\\39609&401436&-476416&536301&-13920&315171&-125696&41796&-1481\\48773&492480&-581120&653265&-9600&364095&-74240&88128&-2005\\503&5832&-8192&9477&-2640&9477&-8192&5832&503\end{pmatrix}\begin{pmatrix}f_n\\f_{n+\frac{1}{6}}\\f_{n+\frac{1}{4}}\\f_{n+\frac{1}{3}}\\f_{n+\frac{1}{2}}\\f_{n+\frac{2}{3}}\\f_{n+\frac{3}{4}}\\f_{n+\frac{5}{6}}\\f_{n+1}\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

(3.1)

After expanding (3.1) in a Taylor series about x_n, we have

$$\bar{c}_0=\bar{c}_1=\bar{c}_2=\cdots+\bar{c}_9=0$$

But

Therefore, as stated in reference [16], the computational block method (2.7) that has been recently formulated demonstrates uniform tenth order.

3.2. Consistency of Computational Method

The computational block method (2.7) demonstrates consistency, as indicated by its order exceeding one $(p\geq 1)$, as referenced in [17].

3.3. Zero Stability of Computational Method

For the computational block method to achieve stability, it is imperative that all roots of the stability polynomial reside within the unit circle in the complex plane, denoted as $|z| < 1$. This criterion guarantees that the numerical solution remains bounded for any initial condition and step size [17]. In the case of the computational block method (2.7) attaining zero-stability, the first characteristic polynomial is provided as follows:

$$\rho(z) = z^6(z-1) = 0$$

$$\Rightarrow z_1 = z_2 = z_3 = z_4 = z_5 = z_6 = 0, z_7 = 1$$

$$\rho(r) = \left| r\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right| = \left| - \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & r & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & r & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & r & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & r & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & r & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & r & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r-1 \end{bmatrix} \right| \quad (3.2)$$

$$= r^7(r-1) = 0$$

thus, solving for r in (3.1), gives

$$r_1 = r_2 = r_3 = r_4 = r_5 = r_6 = r_7 = 0, r_8 = 1$$

Therefore, as stated in [17], the computational block method demonstrates zero-stability.

3.4. Convergence of the Method

Consequently, the computational block method (2.7) converges due to its consistency and zero-stability.

Theorem 3.1

As per reference [17], the linear multistep method converges if and only if it exhibits both consistency and zero-stability.

3.5. Linear Stability

The region of absolute stability of a numerical method is the set of complex values λh for which all solutions of the test problem $y' = -\lambda y$ will remain bounded as $n \to \infty$.

The concept of A-stability according to [18] is discussed by applying the test equation $y^{(k)} = \lambda^{(k)} y$

To yield

$$Y_m = \mu(z) Y_{m-1}, \; z = \lambda h \tag{3.3}$$

Where $\mu(z)$ is the amplification matrix of the form

$$\mu(z) = \left(\xi^0 - z\eta^{(0)} - z^1\eta^{(0)}\right)^{-1}\left(\xi^1 - z\eta^{(1)} - z^1\eta^{(1)}\right) \tag{3.4}$$

The matrix $\mu(z)$ has Eigen values $(0, 0, \cdots, \xi_k)$ where ξ_k is called the stability function.

Thus, the stability function of the method is given by

$$\zeta = -\frac{\begin{pmatrix} 247999497196756695z^8 - 11670731637392660043z^7 + 27128030061143833235z^6 - 51555673500865441394z^5 + \\ 65393261961028561813444z^4 + 658664164691670643931522z^3 + 4301046489378775183096322z^2 - \\ 18744560305848953339904000z + 36690281177719976755200000 \end{pmatrix}}{\begin{pmatrix} 2925595459584000z^8 - 117218858080665600z^7 + 2895169266804326400z^6 - 51394520557604044800z^5 \\ + 675484884472430592000z^4 + 6493698491619997440000z^3 + 43562631298006646784000z^2 \\ - 183451405888599883776000z + 36690281177719976755200000 \end{pmatrix}}$$

4. SAMPLED PROBLEMS

In scientific research, numerical findings and discussions hold significant importance, especially within this study. This section will thoroughly explore the outcomes derived from simulations of the quadratic Riccati differential equation (1.1), offering a detailed analysis and interpretation of these results. To evaluate the effectiveness of our innovative method, we will utilize the computational block method (referred to as Equation (2.7)) to approximate the QRDEs (1.1). Subsequently, a comparison will be made between our results and those obtained by previous researchers, highlighting the superiority of our method over existing ones. The presentation of results in tables will adhere to the specified notation.

ECBM means Absolute Error in Computational Block Method

E[1] means Absolute Error in in [1]

E[11] means Absolute Error in in [11]

E[19] means Absolute Error in in [19]

E[20] means Absolute Error in in [20].

Problem 4.1: Let's consider the Quadratic Riccati Differential Equation in the form

$$y'(x) = 10 + 3y(x) - y^2(x),\ y(0) = 1 \tag{4.1}$$

The exact solution to this equation is expressed as

$$y(x) = -2 + \frac{14e^{7x}}{5 + 2e^{7x}} \tag{4.2}$$

Source: [11, 19].

TABLE 1. Presenting the outcome for Problem 4.1

x	Exact Solution	Computed Solution	ECMB	E[11]	E[19]
0.100	1.12295995501998517310	1.12295995501700859070	2.9766e-12	2.82693e-09	1.46347e-09
0.200	2.33036366723934260660	2.33036366722570309090	1.3640e-11	5.89943e-09	2.99223e-09
0.300	3.35929859139218860340	3.35929859138601452740	6.1741e-12	6.83092e-08	3.49315e-08
0.400	4.07625619989394993700	4.07625619989397232580	2.2389e-14	1.49912e-07	7.66512e-08
0.500	4.50864023794231405830	4.50864023794082590780	1.4882e-12	1.83945e-07	9.40192e-08
0.600	4.74705986375186756050	4.74705986375052733290	1.3402e-12	1.65588e-07	8.46132e-08
0.700	4.87206646548954668230	4.87206646548889184630	6.5484e-13	1.24703e-07	6.37095e-08
0.800	4.93588015111826406050	4.93588015111798985240	2.7421e-13	8.43126e-08	4.30686e-08
0.900	4.96801151790818190370	4.96801151790806507650	1.1683e-13	5.32397e-08	2.71935e-08
1.000	4.98407836223863766150	4.98407836223858426840	5.3393e-14	3.21259e-08	1.64080e-08

Problem 4.2: Let's consider the Quadratic Riccati Differential Equation in the form

$$y'(x) = y^2(x) - 1,\ y(0) = 0 \tag{4.3}$$

The exact solution to this equation is expressed as

$$y(x) = -\tanh(x) \tag{4.4}$$

Source: [1, 11].

TABLE 2. Presenting the outcome for Problem 4.2

x	Exact Solution	Computed Solution	ECMB	E[11]	[1]
0.100	- 0.09966799462495581712	- 0.09966799462495581367	3.4500e-18	2.2921e-12	3.8296e-07
0.200	- 0.19737532022490400074	- 0.19737532022490399467	6.0700e-18	3.1140e-12	3.8296e-07
0.300	- 0.29131261245159090582	- 0.29131261245159089869	7.1300e-18	3.3764e-12	5.7951e-07
0.400	- 0.37994896225522488527	- 0.37994896225522487899	6.2800e-18	3.4242e-12	6.8133e-07
0.500	- 0.46211715726000975850	- 0.46211715726000975459	3.9100e-18	3.3944e-12	7.3394e-07
0.600	- 0.53704956699803528586	- 0.53704956699803528488	9.8000e-19	3.3436e-12	7.6091e-07
0.700	- 0.60436777711716349631	- 0.60436777711716349777	1.4600e-18	3.2949e-12	7.7483e-07
0.800	- 0.66403677026784896368	- 0.66403677026784896651	2.8300e-18	3.2574e-12	7.8257e-07
0.900	- 0.71629787019902442081	- 0.71629787019902442390	3.0900e-18	3.2344e-12	7.8799e-07
1.000	- 0.76159415595576488812	- 0.76159415595576489070	2.5800e-18	3.2265e-12	7.9326e-07

Problem 4.3: Let's consider the Quadratic Riccati Differential Equation in the form

$$y'(x) = 1 - y^2(x),\ y(0) = 0 \tag{4.5}$$

The exact solution to this equation is expressed as

$$y(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \tag{4.6}$$

Source: [11, 20].

TABLE 3. Presenting the outcome for Problem 4.3

x	Exact Solution	Computed Solution	ECBM	E[11]	E[20]
0.100	0.09966799462495581711	0.09966799462495581367	3.4400e-18	1.14908e-14	4.14010e-07
0.200	0.19737532022490400073	0.19737532022490399467	6.0600e-18	6.71685e-14	6.01860e-07
0.300	0.29131261245159090582	0.29131261245159089869	7.1300e-18	1.83353e-13	7.37470e-07
0.400	0.37994896225522488527	0.37994896225522487899	6.2800e-18	3.38618e-13	1.73220e-07
0.500	0.46211715726000975851	0.46211715726000975459	3.9200e-18	4.86111e-13	6.85240e-07
0.600	0.53704956699803528586	0.53704956699803528488	9.8000e-18	5.79870e-13	7.98100e-07
0.700	0.60436777711716349631	0.60436777711716349777	1.4600e-18	5.94858e-13	9.26210e-07
0.800	0.66403677026784896369	0.66403677026784896651	2.8200e-18	5.32796e-13	2.83180e-07
0.900	0.71629787019902442081	0.71629787019902442390	3.0900e-18	4.16112e-13	6.64690e-07
1.000	0.76159415595576488812	0.76159415595576489070	2.5800e-18	2.74558e-13	7.26600e-07

4.1 DISCUSSION OF RESULT

The computational block method was formulated through interpolation and collocation, employing power series polynomials to address Quadratic Riccati differential equations of the form (1.1). A comprehensive analysis of its fundamental properties was conducted, satisfying all requisite conditions for examination. Furthermore, to assess its accuracy and validation, three Quadratic Riccati differential equations were applied to the computational block method during experimentation. The results, presented in Tables 1 to 3, underscore the superiority of the computational block method over existing methodologies proposed by [1, 11, 19, 20]. Specifically, when Quadratic Riccati differential equations (4.1) to (4.3) were subjected to the computational block method (2.7), Table 1 demonstrated its superior performance and efficiency compared to [11, 19]. Similarly, for Problem 4.2, the computational block method (2.7) outperformed [1, 11], as indicated in Table 2. Finally, Table 3 presents the results for Problem 4.3, affirming that the computational block method (2.7) surpasses the approaches of [11, 20].

5. CONCLUSION

This study successfully developed and validated a computational block method for solving Quadratic Riccati Differential Equations (QRDEs). The method was derived using power series interpolation and collocation, and its theoretical foundations were rigorously examined, confirming properties such as high-order accuracy, consistency, zero-stability, and convergence. Numerical experiments conducted on three benchmark problems demonstrated that the proposed method achieves significantly higher accuracy compared to existing techniques in the literature proposed by [1, 11, 19, 20]. The results confirm that the computational block method is a robust, efficient, and reliable numerical tool for solving QRDEs, offering superior precision and convergence behavior. Future work may explore its extension to higher-dimensional or stochastic Riccati-type equations.

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