



## BLOCK BACKWARD DIFFERENTIATION FORMULA FOR CONVECTIVE BOUNDARY CONDITION IN HYDROMAGNETIC HEAT AND MASS TRANSPORT OVER A VERTICAL PLATE

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**ABSTRACT.** This paper examines the solution of boundary condition for the convective surface for a hydromagnetic heat and mass transport over a vertical plate with the use of block method. Similarity solutions were used in converting the partial differential equations regulating the boundary layer into ordinary differential equations. The resultant coupled nonlinear system of ODE was then transformed into a set of equations of the first-order, which was subsequently solved numerically using block backward differentiation formula. The effects of key parameters including magnetic field ( $Ha$ ), Biot number ( $Bi$ ), Grashof number ( $Gr, Gc$ ), and Schmidt number ( $Sc$ ) on velocity, temperature and concentration profiles were examined. The results show that velocity decreases with higher magnetic field but thermal and solutal buoyancy forces; temperature rises with Biot number while Schmidt number reduces concentration boundary thickness. Numerical results demonstrate excellent agreement with existing results, confirming the accuracy of the method. Graphically, the influence of different fluid flow velocity characteristic were highlighted and other physical quantities presented.

### 1. INTRODUCTION

The investigation of mass and heat transport across a moving surface in magnetohydrodynamics (MHD) has important geophysical and technical uses, including nuclear reactor cooling, packed-bed catalytic reactors, increased oil recovery, thermal insulation, and geothermal reservoirs. Additionally, cooling molten liquids that have been stretched into a regulated cooling system is a common step in many procedures in chemical engineering, extrusion of polymers and metallurgy. The stretching rate and cooling liquid selection have a significant impact on the

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final product's mechanical characteristics. Some polymer fluids are commonly utilized as cooling agents because of their superior electromagnetic characteristics, such as Cetane solutions of polyethylene oxide and polyisobutylene. External magnetic fields can modify its flow behaviour to enhance the final product's quality. Sakiadis[22] conducted the first investigation of boundary layer flow in a quiescent ambient fluid caused by a moving plate. A fuller understanding of MHD flow and its applications has resulted from the several researchers who have since investigated different facets of this issue. The research of Makinde[20] and Aziz[13] which was enhanced by Akindele[12] examined the factors that influence energy flow including buoyancy factor, grashof number, magnetic fluid and heat generation absorption. Kaita[14] investigated the natural convection fluid flow on the effect of variable thermal conductivity on heat and mass transfer with chemical reaction, porosity and buoyancy force distribution. Aroloye and Oluwalana[4] expanded on Makinde's work [20] by adding thermal radiation, internal heat generation and a chemical reaction over a moving upright flat plate with a boundary condition that has convective surface to the energy equation and the mass equation, respectively. Adeniyani and Aroloye[10] incorporate joint effect of viscous dissipation, ohmic dissipation with a coupled porous media and magnetic parameter to the boundary layer of a permeable exponentially expanding plate heated by convection exhibits mixed convection flow with internal heat generation, velocity ratio and wall mass flux. Abubakar[1] investigated heat and mass flow through a porous medium with variable thermal conductivity and suction effects, the influence of key dimensionless parameter was highlighted in their work. Omowaye[15] studied Soret and Dufour's impacts. on free convective transport with constant MHD in a viscous dissipating porous media. Aroloye and Oluwalana[5] examined heat and mass transfer of Magnetohydrodynamics flow over a vertical plate in the presence of heat dissipation and thermal Radiation

Given that several boundary layer equations related to the mass and heat transport in fluid problems may provide challenges for analytical resolution, it is imperative to develop numerical methods to get approximate solutions. Various techniques have been employed to analyse the effects of governing parameters in boundary layer equations. Techniques include Adomian Decomposition Method (ADM), Runge-Kutta (RK), Homotopy Perturbation Method (HPM) and Variational Iteration Method (VIM) methods. While HPM and VIM provide accurate results, they are often complex in application (Sobamowo[17][18]). Similarly, the explicit Runge-Kutta method, though widely used, requires large amount of computational Effort requirement (Asifa[6]). This work presents numerical solution for upright flat plate hydromagnetic heat and mass transport with a boundary condition on the convective surface with the use of Block method. A similarity transformation was used in converting the governing equations for the boundary layer, which were first developed as partial differential equations into ODE. Block method was then used in solving these transformed equations. Block method originally proposed by Milne (1953) in order to generate initial values for algorithms that use predictor-correctors and later developed by Rosser (1967)(Akinfenwa[3]) for solving initial value problems was designed to obtain numerical solutions at

multiple points synchronous, block method is well-known for its quick convergence, lower computational cost and great accuracy even for complex systems of equations and unlike traditional methods, it does not require a separate predictor, making it more efficient in terms of both development and execution time. Despite extensive studies using these established methods, as far as the authors are aware, the block method has not yet been widely used in obtaining numerical solution for coupled non linear ODE, given its impressive accuracy in solving a broad range of differential equation like partial differential equations (Akinukawe and Atteh[11], Olaiya[21]), delay differential equations (Rasdi[7], Majid[16]) and stiff differential equations (Adesanya[9], Akinfenwa[3]). This study aims to demonstrate the effectiveness of block method in solving hydromagnetic heat and mass transport problems. The findings displayed in this work further highlight the efficiency and accuracy of the Block method, reinforcing its potential as a robust numerical tool for complex fluid dynamics problems.

## 2. MATERIALS AND METHODS

**2.1. Mathematical Representation:** A consistent, mixed convection transport across a upright flat plate that transfers heat and mass in a laminar, hydro-magnetic manner is analyse. The fluid is said to be electrically conductive and Newtonian with properties influenced solely by fluid density, temperature, and chemical species concentration. Let the  $x$ -axis and the  $y$ -axis be perpendicular to the plate to follow its orientation. The following is an expression for the problem's governing equations under the Boussinesq and viscous layer simplification, assuming that  $T$ ,  $C$   $u$  and  $v$  represent the fluid's temperature, concentration,  $x$ -component of velocity and  $y$ -component of velocity respectively:(see Makinde[20])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (u - U_\infty) + g\beta_T(T - T_\infty) + g\beta_c(C - C_\infty) \quad (2.2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (2.3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (2.4)$$

$\rho$  represent the fluid density, gravitational acceleration represented by  $g$ ,  $\sigma$  represent the conductivity of electricity of the fluid,  $\alpha$  represents the diffusivity of heat,  $D$  represents the diffusivity of mass,  $\beta_T$  is the coefficient of thermal expansion,  $\beta_C$  is the coefficient of solutal expansion,  $\sigma$  is the conductivity of the fluid electricity, and the kinematic viscosity is represented by  $\nu$ . The boundary conditions at the plate surface and deep within the cold fluid can be written as

$$\begin{aligned} u(x, 0) = 0, v(x, 0) = 0, -k \frac{\partial T}{\partial y}(x, 0) &= b[T - T_w(x, 0)], \\ C_w(x, 0) &= Ax_\lambda + C_\infty, \\ u(x, 0) = U_\infty, C(x, \infty) = C_\infty T(x, \infty) &= T_\infty \end{aligned} \quad (2.5)$$

where

$k$  represent the coefficient of the thermal conductivity, at the plate surface  $C_w$  represents the species concentration,  $\lambda$  represents the exponent of the plate surface concentration,  $L$  represent the plate characteristic length and  $b$  represent the coefficient of the plate heat transfer. The continuity was fulfilled by the  $\psi$  which represent the stream function.

$$u = \frac{\partial\psi}{\partial y} \quad v = -\frac{\partial\psi}{\partial x} \quad (2.6)$$

To obtain the similarity solutions of (2.1),(2.2),(2.3) and (2.4), the following similarity variables

$$\eta = y\sqrt{\frac{U_\infty}{vx}}, \quad \psi = \sqrt{vxU_\infty}f(\eta), \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (2.7)$$

are used to transform the partial differential equations (2.2),(2.3) and (2.4) to coupled ordinary differential equations (2.8), (2.9) and (2.10)

$$f''' + \frac{1}{2}ff'' - Ha_x(f' - 1) + Gr_x\theta + Gc_x\phi = 0 \quad (2.8)$$

$$\theta'' + \frac{1}{2}Prf\theta' = 0 \quad (2.9)$$

$$\phi'' + \frac{1}{2}Scf\phi' = 0 \quad (2.10)$$

where

$$Ha_x = \frac{\sigma(B_0)^2x}{\rho U_\infty} \quad Gr_x = \frac{g\beta_T(T_w - T_\infty)x}{(U_\infty)^2}, \quad Gc_x = \frac{g\beta_C(C_w - C_\infty)x}{(U_\infty)^2}, \quad Pr = \frac{v}{\alpha}, \quad Sc = \frac{v}{D}$$

$$\begin{aligned} At \eta = 0, \quad f(\eta) = 0, \quad f'(\eta) = 0, \quad \theta'(\eta) = Bi[\theta(\eta) - 1], \quad \phi(\eta) = 1 \\ At \eta = \infty f'(\eta) = 1, \quad \theta(\eta) = \phi(\eta) = 0 \end{aligned} \quad (2.11)$$

where

$$Bi = \frac{b}{k} \sqrt{\frac{vx}{U_\infty}}$$

Ha for magnetic parameter, Gr and Gc for thermal and solutal Grashof number respectively, Pr stands for prandtl, Sc for Schmidt number, Bi for Biot number.

**2.2. Method of solution:** The block approach was used in this study to solve the MHD fluid flow system of equations, the algorithm introduced by Akinfenwa[3] was adopted. The block method is obtained from the expression

$$Y(x) = -\sum_{j=0}^{q-1} \alpha_j(x)y_{n+j} + h\beta_k(x)f_{n+k} \quad (2.12)$$

where  $\alpha_j(x)$  and  $\beta_k(x)$  are the coefficients of the continuous function. The BDF of  $k = 6$  is generated using this method and  $q = k$ ,  $\psi_j(x) = x^j_{n+i}$ ,  $i = 0, 1, \dots, 6$

which is then evaluated at  $x = x_{n+6}$ , then the additional methods were obtained at point  $x = [x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}, x_{n+5}]$  yielding the following block method

$$\begin{aligned}
 y_{n+6} &= \frac{1}{147} [60hf_{n+6} - 10y_n + 72y_{n+1} - 225y_{n+2} + 400y_{n+3} - 450y_{n+4} + 360y_{n+5}] \\
 f_{n+1} &= \frac{1}{1764h} [24hf_{n+6} - 298y_n - 2235y_{n+1} + 4320y_{n+2} - 2780y_{n+3} + 1290y_{n+4} - 297y_{n+5}] \\
 f_{n+2} &= \frac{1}{2205h} [-15hf_{n+6} + 76y_n - 900y_{n+1} - 1230y_{n+2} + 2840y_{n+3} - 990y_{n+4} + 204y_{n+5}] \\
 f_{n+3} &= \frac{1}{8820h} [60hf_{n+6} - 157y_n + 1395y_{n+1} - 6840y_{n+2} + 400y_{n+3} - 6165y_{n+4} - 963y_{n+5}] \\
 f_{n+4} &= \frac{1}{8820h} [-120hf_{n+6} + 167y_n - 1320y_{n+1} + 4860y_{n+2} - 12560y_{n+3} + 6045y_{n+4} + 2808y_{n+5}] \\
 f_{n+5} &= \frac{1}{8820h} [600hf_{n+6} - 394y_n + 2925y_{n+1} - 9600y_{n+2} + 18700y_{n+3} - 26550y_{n+4} + 14919y_{n+5}]
 \end{aligned} \tag{2.13}$$

The order of accuracy is  $[6, 6, 6, 6, 6, 6]^T$  while error constant is

$$\left[ -\frac{53}{2085}, \frac{18}{1715}, -\frac{167}{20580}, \frac{59}{5145}, -\frac{23}{686}, -\frac{20}{343} \right]^T$$

### 3. RESULT

The temperature of the plate's surface  $\theta(0)$  along with the thermal transfer rate locally at the solid boundary  $\theta'(0)$  for different  $Bi_x$  was compared with those of Aziz[13] and Makinde[20] in order to validate our result in table 3 and we discovered that they have excellent agreement same as those in tables 1 and 2. The rates of mass and heat transfer locally at the plate surface in tables 2 and 3 show an increasing trend as the buoyancy force parameters, convective thermal exchange and the magnetic field parameter increase. Conversely, increasing the Schmidt number, surface mass transfer rate increases also while the rate of heat transmission across the skin and the surface decrease.

TABLE 1. **Finding of  $f''(0)$  and  $\theta(0)$  for various parameters when  $Pr = .72$**

$Bi$	$Gr$	$Gc$	$Ha$	$S_c$	Makinde $f''$	Makinde $-\theta'$	Block BDF $f''$	Block BDF $-\theta'$
0.1	0.1	0.1	0.1	0.24	0.62601	0.07707	0.62599	0.07707
1.0	0.1	0.1	0.1	0.24	0.68213	0.25501	0.68210	0.25501
0.1	0.5	0.1	0.1	0.24	0.72193	0.07760	0.72190	0.07760
0.1	1.0	0.1	0.1	0.24	0.83124	0.07815	0.83120	0.07815
0.1	0.1	0.5	0.1	0.24	1.12789	0.07969	1.12782	0.07969
0.1	0.1	1.0	0.1	0.24	1.66664	0.08143	1.66653	0.08143
0.1	0.1	0.1	1.0	0.24	1.13812	0.07875	1.13800	0.07875
0.1	0.1	0.1	0.5	0.24	2.30247	0.08034	2.30196	0.80347
0.1	0.1	0.1	0.1	0.78	0.59428	0.07671	0.59426	0.07671
0.1	0.1	0.1	0.1	2.62	0.56580	0.07641	0.56578	0.07641

TABLE 2. Computation showing  $\phi(0)$  for the various parameter when  $Pr = 0.72$

$Bi$	$Gr$	$Gc$	$Ha$	$Sc$	Makinde $-\phi'$	Block BDF $-\phi'$
0.1	0.1	0.1	0.1	0.24	0.21859	0.21859
1.0	0.1	0.1	0.1	0.24	0.22169	0.22169
0.1	0.5	0.1	0.1	0.24	0.22381	0.22381
0.1	1.0	0.1	0.1	0.24	0.22937	0.22937
0.1	0.1	0.5	0.1	0.24	0.24852	1.12789
0.1	0.1	1.0	0.1	0.24	0.27339	0.27339
0.1	0.1	0.1	1.0	0.24	0.23452	0.23452
0.1	0.1	0.1	0.5	0.24	0.25077	0.25077
0.1	0.1	0.1	0.1	0.78	0.33955	0.33955
0.1	0.1	0.1	0.1	2.62	0.52053	0.52052

TABLE 3. Computation showing comparison with Aziz[13], Makinde[20] and Block BDF result for  $Ha = 0, Sc = 0.63, Gr = 0, Pr = 0.72, Gc = 0$

$Bi$	Aziz $\theta$	Aziz $-\theta'$	Makinde $\theta$	Makinde $-\theta'$	Block BDF $\theta$	Block BDF $-\theta'$
.05	0.1447	0.0428	0.14466	0.04276	0.14465	0.04276
.10	0.2528	0.0747	0.25275	0.07472	0.25275	0.07472
.20	0.4035	0.1193	0.40352	0.11929	0.40351	0.11929
.40	0.5750	0.1700	0.57501	0.16999	0.57499	0.16999
.60	0.6699	0.1981	0.66991	0.19805	0.66989	0.19805
.80	0.7302	0.2159	0.73016	0.21586	0.73014	0.21586
1.0	0.7718	0.2282	0.77182	0.22817	0.77179	0.22817
5.0	0.9441	0.2791	0.94417	0.27913	0.94414	0.27913
10.0	0.9713	0.2871	0.97128	0.28714	0.97125	0.28714
20.0	0.9854	0.2913	0.98543	0.29132	0.98540	0.29132

#### 4. DISCUSSION

##### Impact of parameter modification on velocity profiles

Figures 1 – 6 reveal how different parameters affect velocity profile within the boundary layer. Figures 1 – 6 show that for all parameter values, the velocity at the surface of the plate starts at zero satisfies the far field boundary requirement by increasing to the undisturbed flow value far from the plate surface. Figure 1 shows how the velocity momentum boundary layer thickness varies when the magnetic field strength increases. Since the fluid velocity is resisted by the drag force produced by the magnetic field, it is now widely known that the magnetic field dampens the velocity field and leads to a decrease in velocity. Here, however, a rise in the magnetic field marginally decrease the fluid's travel toward velocity of the stream flowing away from the surface of the upright flat plate, while increasing fluid's velocity close to the surface of the vertical plate. This indicates

that applying stronger magnetic field can be used to suppress unwanted convection in metallurgical processes such as continuous casting and MHD pumps where precise flow control is required. A comparable pattern of a little rise in fluid speed close to the vertical plate is seen when the parameter for convective heat transfer increases (figure 2). This shows that enhancing convective heat transfer at the surface, for example through surface coatings or extended fins, can significantly improve thermal performance in cooling of electronic devices reactors and solar collectors. Figures 3 and figure 4 illustrate how buoyancy force parameters impact the boundary-layer velocity; In both situations, the fluid accelerates upward close to the upright boundary surface is seen with greater buoyancy force intensity. The flow field slows down to the undisturbed flow velocity further downstream. This trend is important in natural convection cooling of vertical walls in industrial furnaces, nuclear reactors and geothermal systems where buoyancy-driven flows dominate heat transfer. Similarly, the solutal buoyancy drives mass transfer, for instance in distillation, drying and pollutant dispersion. As the Schmidt number rises, Figure 5 illustrates a slight drop in fluid velocity is observed. This reduction in momentum diffusion at higher Schmidt numbers is applicable to chemical separation, drying and polymer processing industries where species transport control is critical.

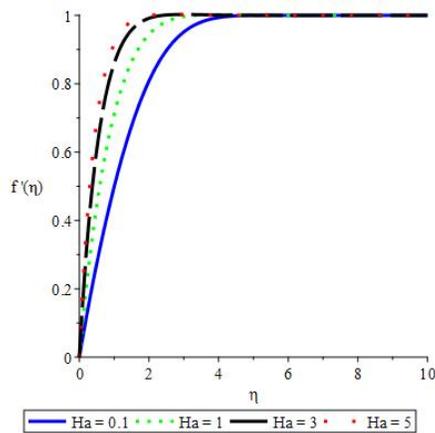


FIGURE 1. Impact of  $Ha$  on Velocity profile when  $Sc = .62$ ,  $Pr = .72$ ,  $Gr = .1$ ,  $Bi = .1$ ,  $Gc = .1$

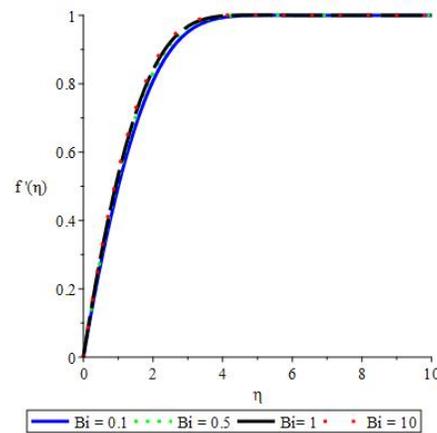


FIGURE 2. Impact of  $Bi$  on Velocity profile when  $Sc = .62$ ,  $Pr = .72$ ,  $Gr = .1$ ,  $Ha = .1$ ,  $Gc = .1$

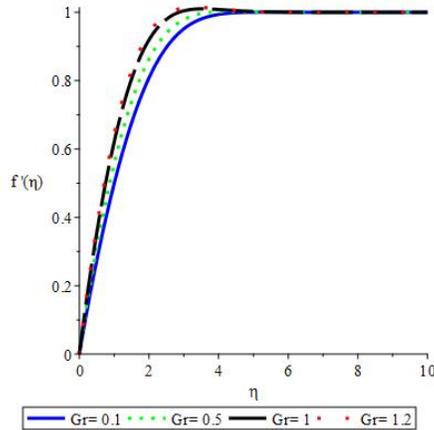


FIGURE 3. Impact of  $Gr$  on Velocity profile when  $Sc = .62, Pr = .72, Ha = .1, Bi = .1, Gc = .1$

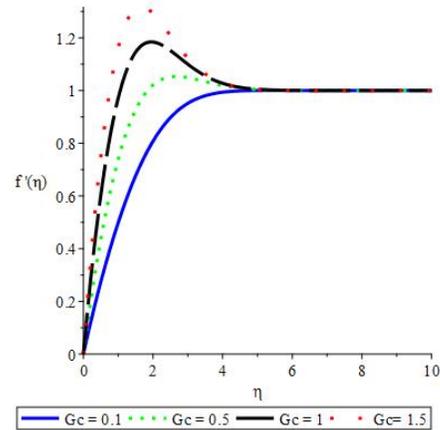


FIGURE 4. Impact of  $Gc$  on Velocity profile when  $Sc = .62, Pr = .72, Gr = .1, Bi = .1, Ha = .1$

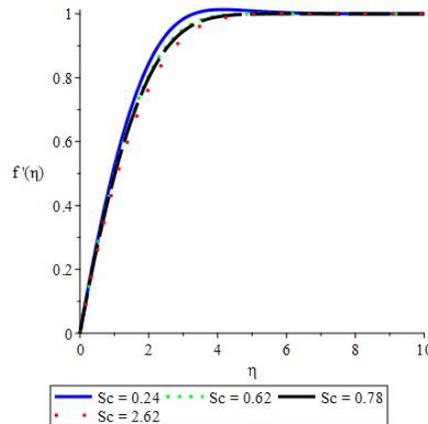


FIGURE 5. Impact of  $Sc$  on Velocity profile when  $Ha = .1, Pr = .72, Gr = .1, Bi = .1, Gc = .1$

### Impact of parameter modification on temperature profiles

Maximum temperature of the fluid is reached at the wall surface and then meets the boundary requirement when it exponentially decreases to the undisturbed flow zero value distant from the plate, which can be seen in figures 6 – 10. It's fascinating to see from these data that when the buoyancy forces and magnetic field intensity increase, the thickness of the heat viscous layer decreases. Additionally, when Schmidt number rises, the fluid temperature rises as well and convective thermal exchange at the wall surface causes thickness of the heat viscous layer to increase. Figure 6 illustrates the impact of  $Ha$  on the temperature profile. This implies that magnetic fields can be effectively employed in thermal

management systems such as MHD generators and cooling channels in high temperature reactors, to regulate temperature distribution. Figure 7 demonstrates the effect of Gr on temperature profile, where buoyancy enhances the thermal layer. This has practical application in solar collectors, building ventilation and geothermal systems where buoyancy induced convection enhance heat transfer efficiency. figure 8, Gc is shown to increase the temperature distribution. This is significant in chemical and process industries, where solutal buoyancy effects can alter heat transfer rates in drying, mixing and reactor operations. figure 9 presents the effect of Schmidt number on the temperature field. this behaviour is relevant is gas absorption, membrane separation and drying technologies, where simultaneous heat and mass transfer is strongly dependent on Schmidt number variations. finally, figure 10 shows that increasing Bi enhances heat transfer at the wall. This highlights the importance of surface convection enhancement in engineering designs such as finned heat exchangers, microelectronic cooling and high efficiency thermal insulation systems.

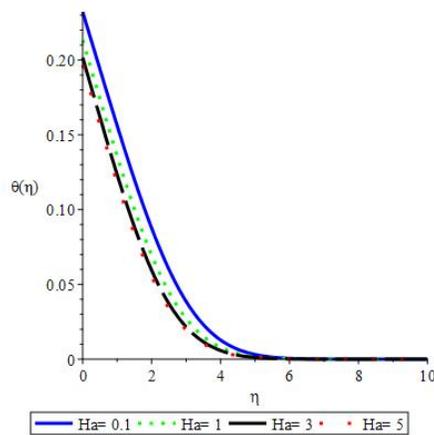


FIGURE 6. Impact of Ha on Temperature profile when  $Sc = .62$ ,  $Pr = .72$ ,  $Gr = .1$ ,  $Bi = .1$ ,  $Gc = .1$

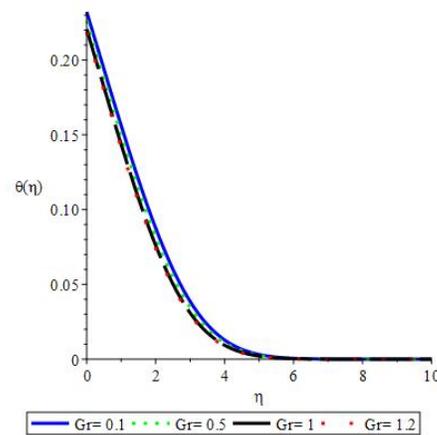


FIGURE 7. Impact of Gr on Temperature profile when  $Sc = .62$ ,  $Pr = .72$ ,  $Gc = .1$ ,  $Ha = .1$ ,  $Bi = .1$

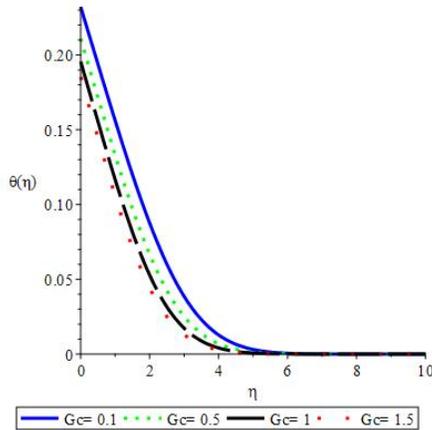


FIGURE 8. Impact of  $G_c$  on Temperature profile when  $Sc = .62$ ,  $Pr = .72$ ,  $Ha = .1$ ,  $Bi = .1$ ,  $Gr = .1$

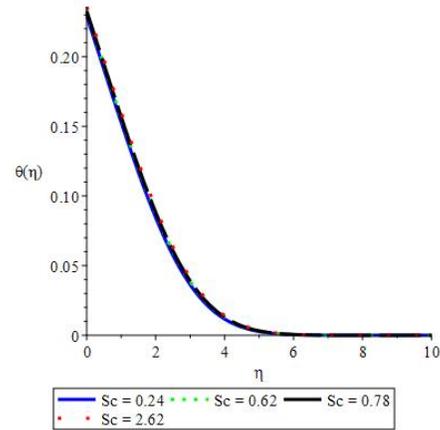


FIGURE 9. Impact of  $Sc$  on Temperature profile when  $G_c = .1$ ,  $Pr = .72$ ,  $Gr = .1$ ,  $Bi = .1$ ,  $Ha = .1$

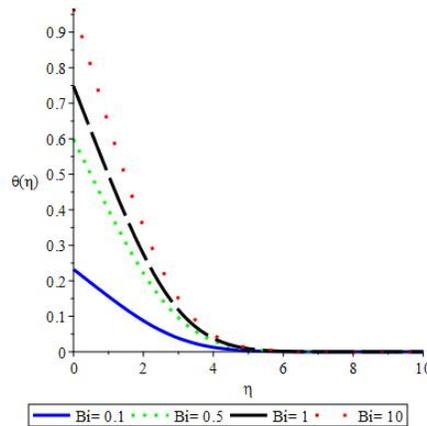


FIGURE 10. Impact of  $Bi$  on Temperature profile when  $Sc = .62$ ,  $Pr = .72$ ,  $Gr = .1$ ,  $Ha = .1$ ,  $G_c = .1$

### Impact of parameter modification on concentration profiles

Figures 11 – 13 reveal the concentration of the chemical species patterns in opposition to the spanwise coordinate  $\eta$  for different physical parameter values within the outermost layer. The solute concentration satisfies boundary condition when it reaches its maximum near the surface of the plate and falls down to zero further from the plate. It is evident from these numbers that as the magnetic field intensity rises, concentration of the viscous layer thickness drops (figure 11). This suggests that magnetic fields can be used to regulate mass transfer rates in metallurgical processes, electrolytic deposition and cooling of electrically conduction fluids. A decreasing concentration thickness is also observes with variations of

Gr (figure 12). This finding is valuable in natural convection mass transfer operations such as evaporation, drying and combustion processes, where buoyancy forces play a dominate role. Similarly, higher Schmidt number (figure 13) leads to reduced concentration boundary layer thickness. This is particular important in chemical process industries involving liquid-gas interactions, polymer solutions and pollutant dispersion, where control of solute diffusion is essential.

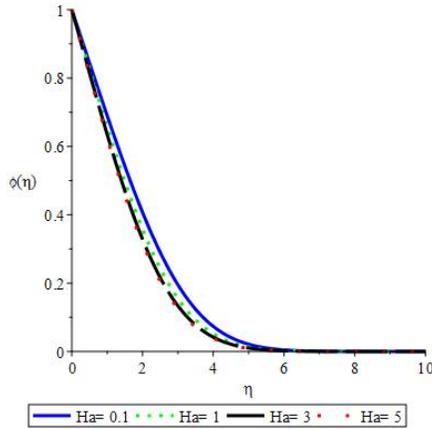


FIGURE 11. Impact of  $Ha$  on concentration profile when  $Sc = .62, Pr = .72, Gr = .1, Bi = .1, Gc = .1$

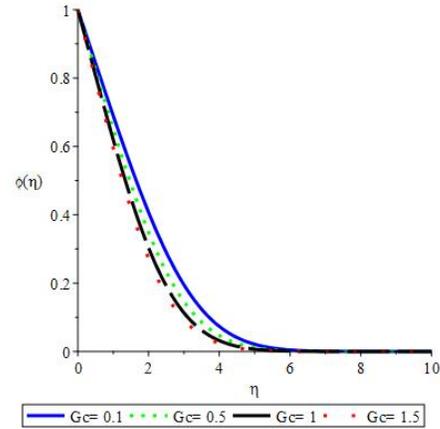


FIGURE 12. Impact of  $Gc$  on concentration profile when  $Sc = .62, Pr = .72, Gr = .1, Ha = .1, Bi = .1$

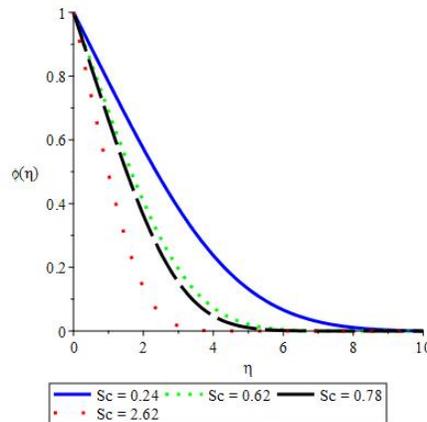


FIGURE 13. Impact of  $Sc$  on concentration profile when  $Ha = .1, Pr = .72, Gr = .1, Bi = .1, Gc = .1$

## 5. CONCLUSION

In the paper, numerical solution for hydromagnetic heat and mass transfer across a vertical plate with convective boundary condition was presented using

the block method of Akinfenwa[3]. The results demonstrate excellent agreement with existing literature, confirming the accuracy and reliability of the method. The study shows that velocity is suppressed under stronger magnetic fields, buoyancy parameters enhance fluid motion near the surface, Biot and Schmidt numbers strongly influence heat and mass transfer characteristic. These findings are of significance in engineering processes such as nuclear reactor cooling, metallurgical casting, electronic device thermal management and MHD generator design. The robustness of the block method makes it a useful tool for analysing similar transport problems where accuracy and efficiency are required. Future research may extend this approach to unsteady and three-dimensional problems thereby strengthening its potential industrial applicability.

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**Authors Contributions.** Each author contributed equally to this work

**Authors' Conflicts of interest.** The authors declare no conflict of interest

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