



POLICE PATROL OPTIMIZATION: A PROFICIENCY-AWARE MODEL FOR RAPID RESPONSE TO INCIDENTS

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ABSTRACT. This study introduces the *Crime Neutralization Model* (CNM), an optimization based framework designed to improve the strategic deployment of police patrol teams. The CNM addresses critical limitations in traditional rapid response systems by incorporating three, often neglected operational factors: (1) variation in patrol team proficiencies, (2) crime-type-specific resource demands and (3) scenario-dependent severity levels. A discrete mathematical programming approach is employed to minimize the expected weighted response distance, while ensuring that all incidents are covered by suitably proficient teams within defined response radii. The model is validated using synthetic urban crime data, where results demonstrate that 10 patrol teams operating within a 1.5 km response radius achieve a 45.29% improvement in travel distance efficiency. Sensitivity analysis yields two key insights: first, optimal deployment strategies remain consistent across increasing response distances, indicating robustness; second, performance does not scale linearly with team count, most notably, a deployment of 12 teams unexpectedly results in reduced efficiency. With computation times ranging from 4 to 12 seconds, the CNM offers a practical and adaptive tool for police departments to enhance resource allocation in alignment with both spatial constraints and operational capabilities.

1. INTRODUCTION

Effective policing is fundamental to ensuring public safety and responding promptly to emergencies [1]. To optimize emergency response capabilities, police departments have turned to data-driven patrol deployment models that prioritise minimizing response times [2]. These models focus on strategic resource allocation to enable swift intervention during critical incidents.

2010 *Mathematics Subject Classification.* Primary: 90B80. Secondary: 90C10.

Key words and phrases. police patrol deployment, facility location, crime neutralization, resource allocation, proficiency-constrained patrol allocation, heterogeneous team utilization, Crime-severity combination.

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Submitted: June 27, 2025. Revised: September 16, 2025. Accepted: September 24, 2025.

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Optimization techniques have emerged as valuable tools for enhancing rapid response capabilities. By employing models such as the Facility Location Problems (FLP) and the Vehicle Routing Problem (VRP), researchers have developed methods to optimize resource allocation and minimize response times [3]. These adapted law enforcement models aim to strategically position patrol teams for maximum responsiveness [3].

However, a critical review of the literature reveals that current optimization models often fail to fully capture the complex, multi-faceted reality of police response. Specifically, they frequently rely on three simplifying assumptions that limit their practical effectiveness: (1) treating patrol teams as homogeneous units with uniform capabilities, (2) aggregating crime types and severity levels into a single, generic demand metric, and (3) focusing solely on spatial coverage without considering the tactical suitability of a team to handle a specific incident. These oversights neglect the stochastic nature of emergency response needs, the specific resource requirements for each crime-severity combination, and the varying proficiency of patrol teams. Consequently, existing models may yield deployment plans that are spatially efficient but operationally infeasible, potentially increasing the risk of patrol teams being overpowered by criminals.

This research, therefore, develops the *Crime Neutralization Model* (CNM), a novel optimization framework that advances the state-of-the-art by integrating three critical, yet often overlooked operational factors: (1) *team heterogeneity*, by explicitly modelling the unique proficiency of each patrol team across diverse crime types and severity levels; (2) *incident-resource specificity*, by incorporating scenario-dependent resource requirements that define the minimum proficiency needed to neutralize an incident; and (3) *stochastic demand*, by accounting for the probability of each crime-severity combination occurring at a given location.

By moving beyond traditional coverage models to this integrated proficiency-aware framework, the CNM ensures that response plans are not just fast, but also effective and tactically efficient. This directly addresses a key gap between theoretical resource allocation and operational reality in reactive policing.

The remainder of this paper is structured as follows. Section 2 reviews the relevant literature. Section 3 presents the model and its underlying assumptions. Section 4 details the experimental results, and Section 5 discusses their implications. Section 6 outlines the study's limitations and suggests directions for future research. Finally, Section 7 concludes the study, and Section 8 provides acknowledgements.

2. REVIEW OF RELATED STUDIES

Police location-allocation planning plays a vital role in ensuring rapid response to incidents by strategically deploying patrol teams [4]. While crime prevention remains a central goal, an equally urgent priority is the swift neutralization of threats once a crime occurs or is about to occur. For a police response team to respond effectively to incidents, their positioning while not attending to any incident can be optimized [5]. Therefore, Mathematicians, specifically Operational Researchers became involved and began applying Facility Location-Allocation

models such as the p-Median model, Set Covering Location model and the Maximal Covering Location model to develop police patrol deployment models that support rapid incident response.

Early work by Su *et al.* [6] introduced a two-level police patrol optimization model that tackled three critical needs: emergency response, unpredictable patrol routes and priority-based area coverage. Their model combined sector design with hierarchical response time requirements, using a branch-and-bound approach to reduce patrol sectors by 25 percent while maintaining response efficiency. Despite its strengths, the model assumed homogeneous patrol capabilities and made simplified assumptions about road and traffic conditions, limiting its real-world applicability.

Building on such foundational work, Chow *et al.* [7] applied the p-Median model to specifically minimize response times, marking a shift toward incident-driven deployment. However, their design assigned only one patrol team per area, disregarding differences in team efficiency or incident severity. Meanwhile, Guedes *et al.* [8] advanced the field with a dispatch optimization system that used multi-objective evolutionary algorithms (MOEA) alongside real crime data from a Brazilian city. Their model effectively reduced average response times by 20 percent and achieved near-perfect priority call attendance. Nevertheless, it failed to account for patrol team heterogeneity or crime-specific severity distinctions, factors that are increasingly crucial in modern policing.

A significant parallel evolution in patrol optimization has been the rise of predictive policing models, which seek to anticipate criminal activity rather than merely react to it. Ground-breaking work by Mohler *et al.* [9] introduced crime forecasting through self-exciting point process models, treating crime like an epidemic that spreads from prior incidents. This was operationalized in the work of Isafiade *et al.* [10], which used historical crime data to generate daily patrol recommendations. However, predictive models often operated in a silo, separate from resource allocation algorithms. They generated risk maps but did not directly optimize how or where to deploy patrol units in response to those predictions, creating a gap between forecasting and execution. Furthermore, it was earlier noted that these models have faced substantial ethical scrutiny regarding the potential perpetuation of biased enforcement patterns embedded in historical data [11].

A more data-driven approach came from the work of Mukhopadhyay *et al.* [12], who proposed a bi-level stochastic optimization framework combining real-time crime forecasting with adaptive routing. Using survival analysis-based crime prediction models and iterative decomposition, they achieved faster and more accurate results than existing methods. Yet, like earlier models, theirs assumed uniform patrol team proficiency and simplified incident classification, limiting its adaptability to varied urban crime scenarios.

Similarly, Chase *et al.* [13] proposed a spatio-temporal Mixed Integer Linear Programming model that ensured response time is minimized, by incorporating real police data and machine learning for travel time estimation. Their model

achieved resource savings while maintaining service levels, but again, the assumption of homogeneous patrol teams and fixed urgency levels limited its flexibility in responding to various incident types.

Moving beyond static deployment, a growing body of research focuses on dynamic patrol models that adapt to changing conditions in real or near-real time. These models often draw from advanced artificial intelligence, treating patrol as a sequential decision-making problem. For instance, Chen *et al.* [1] have employed sophisticated Multi-Agent Reinforcement Learning (MARL) frameworks to generate adaptive patrol strategies that continuously learn and react to criminal activity. Similarly, other approaches have utilized game theory to model the strategic interplay between police and adversaries, optimizing randomized patrol routes to prevent predictable patterns [14] and [15]. While these methods offer powerful adaptability, their "black-box" nature can make them difficult to interpret and implement within the structured command protocols of real police departments.

Most recently, Dewinter *et al.* [2] implemented a p-Median-based deployment strategy for police vehicles in Antwerp. While focused on both preventive and reactive policing, the model limited each zone to one vehicle, disregarding differences in demand across zones an oversight that could hamper response effectiveness in high-crime areas. Price & Curtin [16] employed the Maximal Covering Location Problem Model (MCLPM) combined with integer programming to minimize police response time. While the model effectively addresses the goal of rapid deployment, it assumes that each hotspot has only one type of demand and fails to consider the varying proficiencies of patrol teams in handling different types of crimes.

Despite the substantial progress made in police location-allocation modelling for rapid incident response, existing models often rely on simplifying assumptions that limit their practical effectiveness. A recurring gap across the reviewed studies is the assumption of homogeneous patrol teams and the failure to differentiate between varying levels of incident severity. These oversights weaken the adaptability and operational realism of such models in complex urban settings. Building on this foundation, the model proposed in this study seeks to address these limitations by explicitly incorporating the incident-severity combinations and the proficiency of patrol teams in handling each specific combination. By aligning team capabilities with the nature and urgency of emerging incidents, the model aims to enhance response precision, reduce operational delay and strengthen the overall effectiveness of police deployment strategies. This contribution advances the field toward more responsive, context-aware and efficient police patrol systems.

3. MODEL FORMULATION

This section presents the Crime Neutralization Model (CNM), a stochastic optimization framework designed to efficiently allocate limited police patrol resources across urban areas. The model ensures rapid response to incidents while adhering to critical operational capacity constraints.

The CNM introduces key enhancements to improve the operational effectiveness of police patrol teams. It accounts for patrol team proficiency constraints and their varying abilities to handle different types of crimes under different severity levels. By jointly considering the type and severity of incidents, along with the proficiency of patrol teams in managing these under various scenarios, the CNM offers a more structured and adaptive framework for crime neutralization. The remainder of this section is organized as follows. Section 3.1 outlines the fundamental assumptions of the model. Section 3.2 introduces the mathematical elements, including sets, parameters, and decision variables. Finally, Section 3.3 presents a conceptual benchmarking against existing rapid response deployment models.

3.1. Model Assumptions. The CNM is built upon a set of core assumptions that reflect the operational and logistical realities of urban police patrol deployments. These assumptions are categorized into foundational and enhanced realism assumptions.

Foundational Assumptions:

- i. Patrol deployment is discrete, with a fixed number of patrol teams available at the time of deployment.
- ii. Each patrol team can only respond to incidents within a defined maximum response distance from its assigned patrol location.
- iii. Each potential incident (a unique crime-severity combination at a location) is attended to by a single patrol team.
- iv. Each patrol location can accommodate at most one patrol team.
- v. The number of available patrol teams is less than the number of patrol locations.
- vi. The number of patrol locations is fewer than the number of incident locations (hotspots).

Assumptions for Enhanced Realism:

- i. Patrol teams are heterogeneous, possessing varying levels of proficiency in handling different crime types under various severity levels.
- ii. Each incident requires a specific level of police resources, determined by its crime type and severity level.
- iii. The probability of a specific crime-severity combination occurring at a location is known.
- iv. Deployment decisions are scenario-dependent, adapting to both the probability of crime occurrence and the proficiency of available patrol teams.

These assumptions collectively ensure the model is both computationally tractable and aligned with the strategic requirements of reactive policing, providing a responsive and efficient framework for patrol deployment.

3.2. Mathematical Elements of the Model. To facilitate a clear understanding of the model, we define the following sets, parameters and decision variables:

1. Sets:

- i. Set of demand locations (incident locations or hotspots), $I = \{1, 2, \dots, n\}$, where n is the number of potential incident locations.

- ii. Set of patrol locations, $J = \{1, 2, \dots, m\}$, where m is the number of patrol locations.
- iii. Set of crime categories, $C = \{c_1, c_2, \dots, c_k\}$, where k is the number of crime categories (e.g., violent crime, property crime, kidnapping, terrorism, etc.).
- iv. Set of possible scenarios (severity levels), $S = \{s_1, s_2, \dots, s_u\}$, where u is the number of scenarios (e.g., very severe, severe, minor).
- v. Set of patrol teams, $T = \{1, 2, 3, \dots, v\}$, where v is the number of patrol teams.

2. Parameters:

- i. Total Number of patrol teams to be deployed, K , which is the number of patrol teams available at the time of deployment.
- ii. The shortest distance $d_{i,j}$ between patrol location j and incident location i , the Euclidean distance between their geometric coordinates. That is, if the coordinates of hotspot i are (z_i, y_i) and the coordinates of patrol location j is (z_j, y_j) , then, $d_{i,j}$ is defined as

$$d_{i,j} = \sqrt{(z_j - z_i)^2 + (y_j - y_i)^2}.$$

- iii. The maximum response distance, r , which is the maximum distance a patrol team can travel from its patrol location.
- iv. Response parameter, $\lambda_{i,j}$, defined by:

$$\lambda_{i,j} = \begin{cases} 1, & \text{if } d_{i,j} \leq r, \\ 0, & \text{otherwise.} \end{cases}$$

- v. Level of police resources required at incident location i for crime c under scenario s : $L_i^{c,s} \geq 0$.
- vi. Probability of crime category c under scenario s occurring at incident location i , $Q_i^{c,s}$, where

$$\sum_{c \in C} \sum_{s \in S} Q_i^{c,s} = 1 \quad \forall i \in I$$

- vii. Proficiency of patrol team t for crime c under scenario s : $P_t^{c,s} \geq 0$.

3. Decision Variables:

- i. Binary decision variable x_j , defined by:

$$x_j = \begin{cases} 1, & \text{if patrol location } j \text{ is assigned a patrol team,} \\ 0, & \text{otherwise.} \end{cases}$$

- ii. Binary decision variable $y_{i,j}^{c,s}$, defined by:

$$y_{i,j}^{c,s} = \begin{cases} 1, & \text{if incident at location } i \text{ is to be responded to by a} \\ & \text{patrol team at location } j \text{ for crime category } c \\ & \text{under scenario } s, \\ 0, & \text{otherwise.} \end{cases}$$

iii. Binary decision variable $z_{j,t}$ defined by:

$$z_{j,t} = \begin{cases} 1, & \text{if patrol team } t \text{ is assigned to patrol location } j, \\ 0, & \text{otherwise.} \end{cases}$$

Using the defined mathematical elements, we now present the complete CNM. Formulated based on the outlined assumptions and operational requirements, the model consists of an objective function that minimizes the expected weighted response distance and a set of constraints that ensure solution feasibility. The complete model is presented as follows:

$$\text{Minimize: } Z = \sum_{i \in I} \sum_{j \in J} \sum_{c \in C} \sum_{s \in S} L_i^{c,s} \cdot Q_i^{c,s} \cdot d_{i,j} \cdot y_{i,j}^{c,s}, \quad (1)$$

$$\text{Subject to: } \sum_{j \in J} x_j \leq K, \quad (2)$$

$$\sum_{t \in T} z_{j,t} = x_j, \quad \forall j \in J, \quad (3)$$

$$\sum_{t \in T} z_{j,t} \leq 1, \quad \forall j \in J, \quad (4)$$

$$y_{i,j}^{c,s} \leq x_j \lambda_{i,j}, \quad \forall i \in I, \forall j \in J, \forall c \in C, \forall s \in S, \quad (5)$$

$$L_i^{c,s} y_{i,j}^{c,s} \leq \lambda_{i,j} \sum_{t \in T} P_t^{c,s} z_{j,t}, \quad \forall i \in I, j \in J, c \in C, s \in S, \quad (6)$$

$$\sum_{j \in J} y_{i,j}^{c,s} = 1, \quad \forall i \in I, c \in C, s \in S, \quad (7)$$

$$x_j \in \{0, 1\}, \quad \forall j \in J \quad (8)$$

$$z_{j,t} \in \{0, 1\}, \quad \forall j \in J, t \in T, \quad (9)$$

$$y_{i,j}^{c,s} \in \{0, 1\}, \quad \forall i \in I, j \in J, c \in C, s \in S. \quad (10)$$

In the above model, as explained earlier, the objective function (1) minimizes the expected weighted response distance from patrol locations to incident locations. Constraint (2) is the resource constraint, which ensures that the number of deployed patrol teams does not exceed the available number K at the time of deployment. Constraint set (3) governs patrol team assignment. It ensures that a patrol team can only be deployed to a location that has been activated. Constraint set (4) enforces unique team assignment, allowing at most one team to be deployed to an activated patrol location. Constraint set (5) ensures response feasibility. It guarantees that an incident of category c and severity s at location i can be responded to from location j only if a patrol team is deployed at j and i lies within the response radius of j determined by $\lambda_{i,j}$. Constraint set (6) enforces proficiency requirements. It ensures that an incident of type c and severity s at location i can be responded to by a team deployed at j only if i is within the response distance of j (as determined by $\lambda_{i,j}$) and that the team t has the required proficiency to neutralize the incident. Constraint set (7) ensures a unique incident-response assignment. It guarantees that each incident of any crime-severity combination is responded to from only one patrol location. That

is, only one team can respond to each crime-severity combination at a time. This constraint prevents resource duplication and ensures that all incidents are attended to efficiently. Constraint set (8) is a binary constraint indicating whether a patrol location is activated or not. Constraint set (9) is also a binary constraint indicating whether patrol team t is assigned to location j or not. Finally, constraint set (10) is also a binary constraint indicating whether an incident of category c and severity s at location i is to be responded to from patrol location j .

Having presented the CNM to address critical complexities like resource constraints, heterogeneous team proficiencies and incident-severity combinations, we first situate our model within the existing literature. Before proceeding to empirical validation, it is essential to conceptually benchmark the CNM against established rapid response patrol models. This comparison highlights the novel contributions of our framework and provides a theoretical foundation for expecting superior performance. Following this conceptual analysis, we evaluated the model's practical effectiveness through rigorous empirical testing and sensitivity analysis.

3.3. Conceptual Benchmarking with Existing Rapid Response Patrol Models. To clarify the distinct advantages of the proposed CNM formulation over approaches commonly found in the literature, Table 1 provides a side-by-side comparison of key capabilities. While no numerical benchmarking is conducted due to structural and dimensional differences in the model's scope, this conceptual mapping highlights the additional operational realism embedded in our formulation.

Table 1: Comparison of key features between existing models and the proposed CNM formulation

Feature/Capability	Existing Models in Literature	Proposed CNM
Primary Objective	Minimize response distance	Minimize response distance while also integrating scenario-based constraints, proficiency matching and physical feasibility
Incident-Patrol Location Coverage Relation	Binary coverage	Distance-based coverage enforced via a maximum response distance r and Proficiency-graded feasibility checks via constraint (6)

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Feature/Capability	Existing Models in Literature	Proposed CNM
Crime Scenario Handling	Single-type or aggregated crime demand	Multiple crime categories with scenario-specific probabilities
Team Proficiency	Often ignored or uniform	Explicit proficiency parameter ($P_t^{c,s}$) defined per team–crime type–scenario combination
Resource Needs	Aggregate or fixed	Scenario-specific resource requirements linked to team deployment
Deployment Constraints	Limited to team–location assignment	Includes mutual exclusivity, capacity and scenario-aware feasibility
Output Detail	Basic deployment plan	Deployment plan plus coverage validation, feasibility flags and scenario-by-scenario performance
Scalability	Moderate (fewer sets and parameters)	Higher complexity with additional sets, enabling more realistic decision-making
Applicability	Generalized coverage problems	Tailored to law enforcement patrol planning under complex operational realities

4. RESULTS

This section presents the empirical findings from testing the Crime Neutralization Model (CNM) on a synthetic urban setting. The results demonstrate the model’s performance in optimizing patrol deployment, its computational efficiency and its behaviour under varying operational parameters.

4.1. Experimental Set-up. To evaluate the CNM, a numerical experiment was conducted using a synthetically generated dataset. Coordinate positions were randomly assigned to 25 red dots, representing incident locations (hotspots) and 15 blue squares representing patrol locations, within a 70 km² urban area, designed as a square grid measuring approximately 8.4km per side. The spatial configuration of these points is shown in Figure 1. Three crime types (Theft,

Assault, Kidnapping) and three severity levels (Minor, Severe, Very Severe) were used. Since real-world data on police resource requirements and team proficiencies were unavailable, the values for the resource requirement parameter $L_i^{c,s}$ and the proficiency parameter $P_t^{c,s}$ were generated synthetically to illustrate the models mechanics and ensure a feasible solution space.

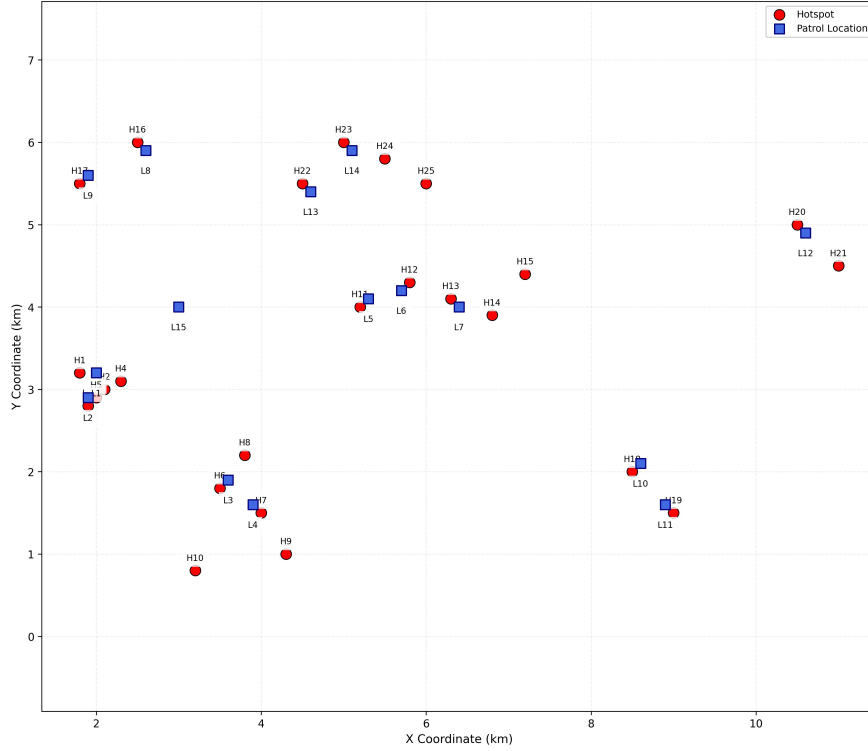


FIGURE 1. Spatial Distribution of Hotspots and Patrol Locations Within a 70 km².

The model was implemented in Python using PuLP library, and solved using the CBC (Coin-or Branch and Cut) solver, an exact optimization approach for integer programming problems. The baseline scenario deployed 10 heterogeneous patrol teams within a maximum response distance (r) of 1.5 km. The key performance outcomes are summarized in Table 2.

The model achieved full coverage of all 225 crime-scenario combinations. The optimal deployment plan, which strategically assigns teams to locations based on both spatial proximity and proficiency matching is detailed in table 3 and visualized in figure 2.

Table 3: Optimal deployment plan of the CNM.

Team	Assigned Location	Hotspots handled	Crime-severity handled
T ₁	L ₁₁	H ₁₈ , H ₁₉	All crimes-severity combinations

Continued on next page

Team	Assigned Location	Hot Spots handled	Crime-severity handled
T ₂	L ₁₃	H ₂₂	Theft and Assault (all severity levels); Kidnapping (Severe and Very Severe)
T ₃	L ₁₂	H ₂₀ , H ₂₁	All crimes-severity combinations
T ₄	L ₂	H ₁ –H ₅	All crimes-severity combinations
T ₅	L ₈	H ₁₆ –H ₁₇	H ₁₆ : Theft (Very Severe), Assault and Kidnapping (all severity combinations), H ₁₇ : Kidnapping/Minor
T ₆	L ₇	H ₁₃ –H ₁₅	All crimes-severity combinations
T ₇	L ₄	H ₆ –H ₁₀	All crimes-severity combinations
T ₈	L ₉	H ₁₆ –H ₁₇	Theft (Minor/Severe at H ₁₆); All crimes-severity combinations at H ₁₇ (except Kidnapping/Minor)
T ₉	L ₁₄	H ₂₂ –H ₂₅	Kidnapping (Minor at H ₂₂); All crimes-severity combinations at H ₂₃ –H ₂₅
T ₁₀	L ₆	H ₁₁ , H ₁₂	All crimes-severity combinations

4.2. Sensitivity Analysis. A sensitivity analysis was conducted to evaluate the model's performance under varying parameters. Table 4 presents the results for different numbers of available teams and response distances.

TABLE 2. Summary of CNM Baseline Results (10 teams, $r = 1.5$ km).

Component	Value
Number of Teams Available	10
Acceptable response distance (km)	1.5
Incidents Covered	all (225)
Objective Function Value (km)	292.2601
Total Travel Distance (km)	90.09
Maximum Possible Travel Distance (km)	198.92
Travel Distance Percentage	45.29%
Number of Teams Used	10
Computation Time (seconds)	8.29
Memory Usage (Peak, MB)	29.99

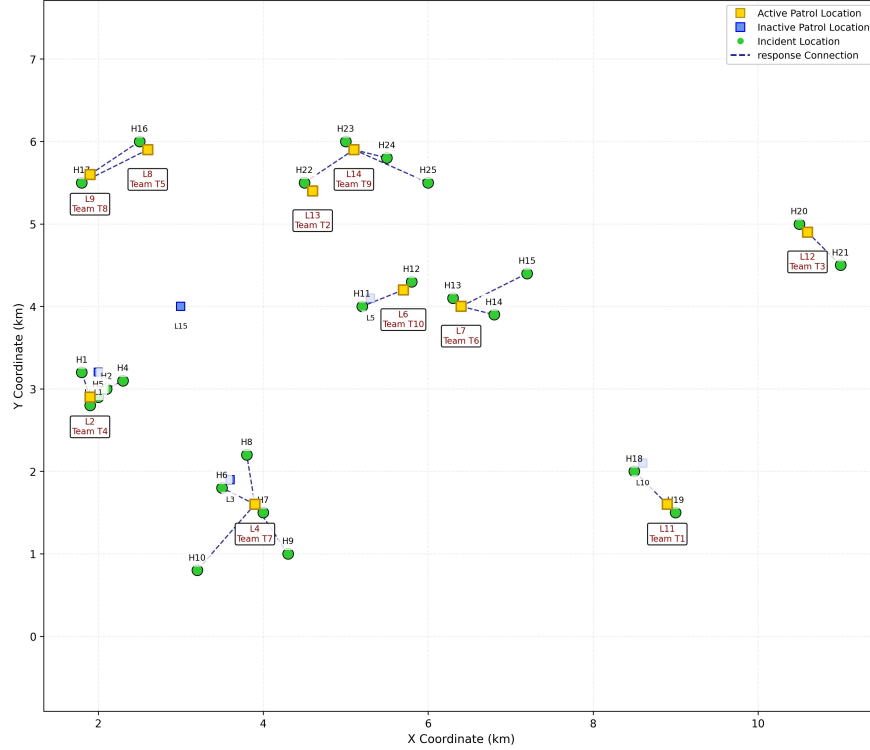


FIGURE 2. Optimal Deployment of 10 Patrol Teams With 1.5km Response Distance.

Table 4: CNM Performance Metrics under Parameter Variation

No. of teams	Response distance (km)	Total travel distance/Total possible travel distance (km)	Total Travel distance %	Objective function value (km)	NO. of Teams used	Time (sec)

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No. of teams	Response distance (km)	Total travel distance/Total possible travel distance (km)	Total travel distance %	Objective function value (km)	NO. of Teams used	Time (sec)
10	1.5	90.09/198.92	45.29	292.2601	10	8.29
10	2.0	90.09/319.88	28.17	292.2601	10	8.01
10	2.5	90.09/403.51	22.33	292.2601	10	9.87
10	3.0	90.09/535.63	16.82	292.2601	10	4.68
12	1.5	100.75/198.92	50.65	315.6306	12	11.52
12	2.0	100.75/319.88	31.50	315.6306	12	10.62
12	2.5	100.75/403.51	24.97	315.6306	12	8.68
12	3.0	100.75/535.63	18.81	315.6306	12	9.26
14	1.5	87.66/198.92	44.07	278.8098	14	8.47
14	2.0	87.66/319.88	27.40	278.8098	14	9.77
14	2.5	87.66/403.51	21.72	278.8098	14	10.87
14	3.0	87.66/535.63	16.36	278.8098	14	10.26
15	1.5	84.72/198.92	42.59	270.6370	14	8.19
15	2.0	84.72/319.88	26.48	270.6370	14	8.63
15	2.5	84.72/403.51	20.99	270.6370	14	9.55
15	3.0	84.72/535.63	15.82	270.6370	14	12.13

Key findings from the sensitivity analysis are as follows:

- i. For a fixed number of teams, the objective function value and total travel distance remained constant across all tested response distances (1.5 km to 3.0 km).
- ii. The travel distance percentage decreased as the response distance increased for all team configurations.
- iii. A non-monotonic relationship was observed when varying the number of teams: performance worsened with 12 teams compared to 10 teams, before improving with 14 and 15 available teams.
- iv. Increasing the number of teams does not necessarily require the model to deploy all available resources. For example, when 14 teams were available, the model deployed all 14. However, when 15 teams were available, the model still deployed only 14, yet the objective function improved due to the increased flexibility in selecting the optimal subset. This outcome, permitted by constraint (2), further demonstrates the correctness of the formulation.
- v. Computation times remained low, between 5 and 12 seconds, for all scenarios.

These trends are further illustrated in the following figures, which visualize the relationship between response distance and key performance metrics for different team counts.

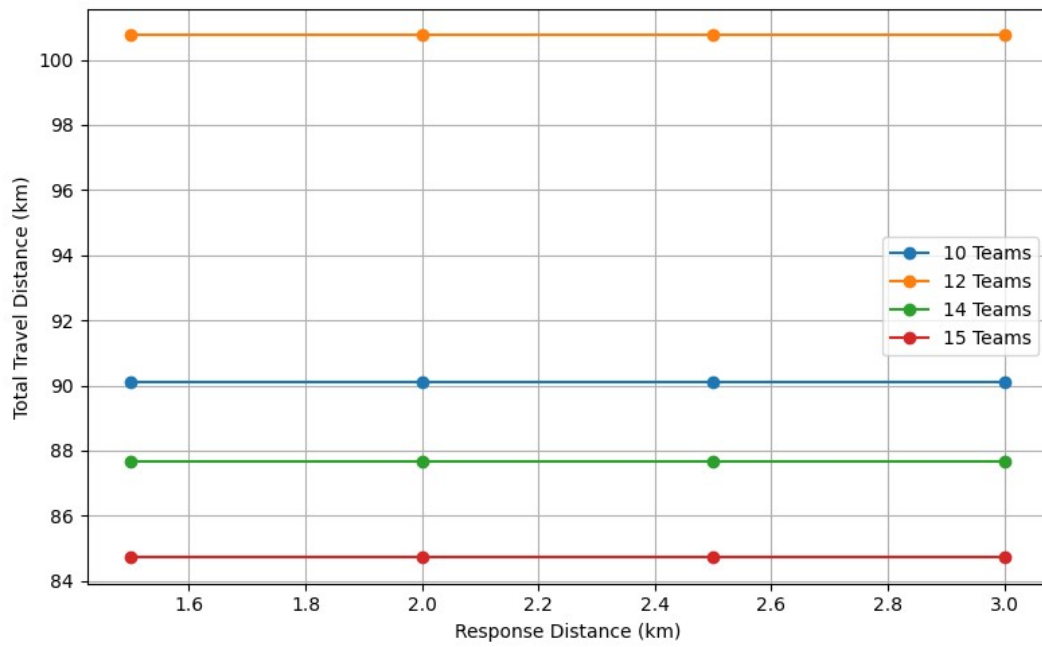


FIGURE 3. Total Travel Distance Against Response Distance.

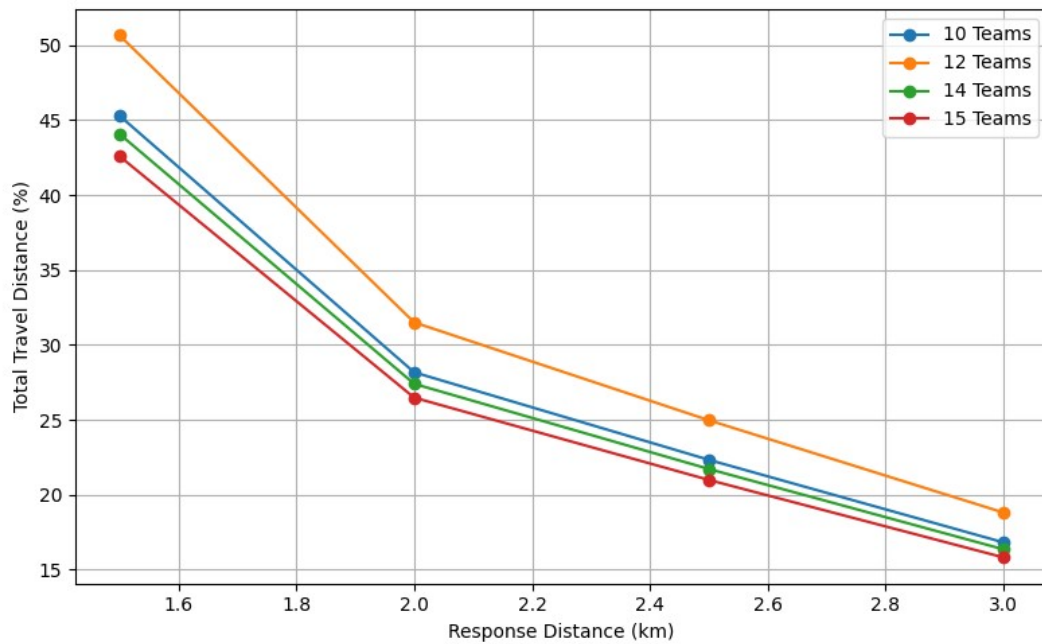


FIGURE 4. Percentage of Total Travel Distance Against Response Distance.

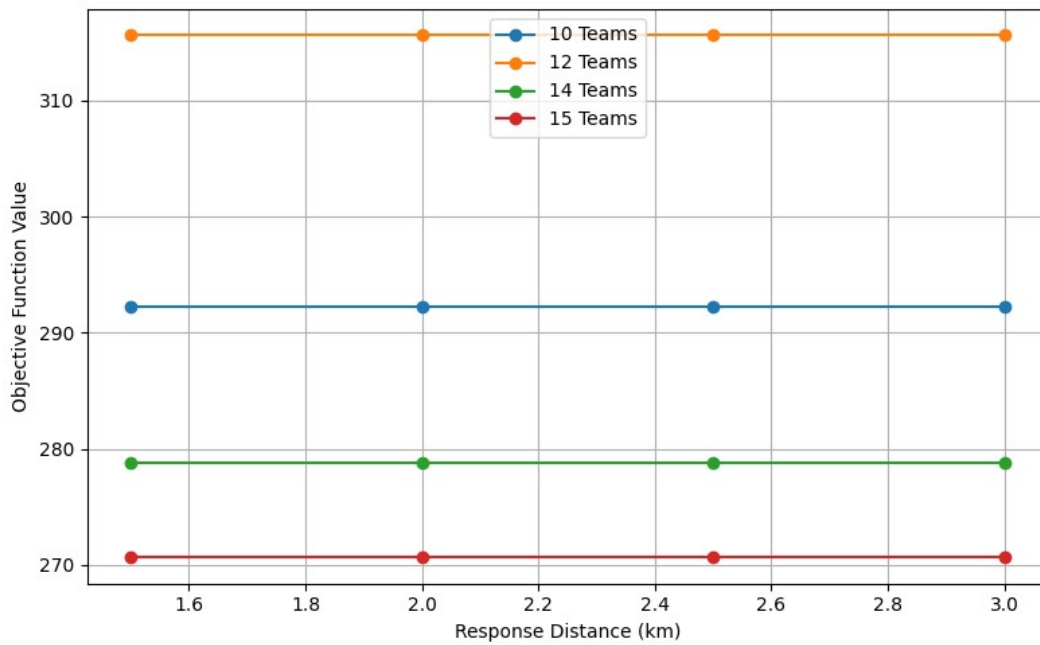


FIGURE 5. Objective Function Value Against Response Distance.

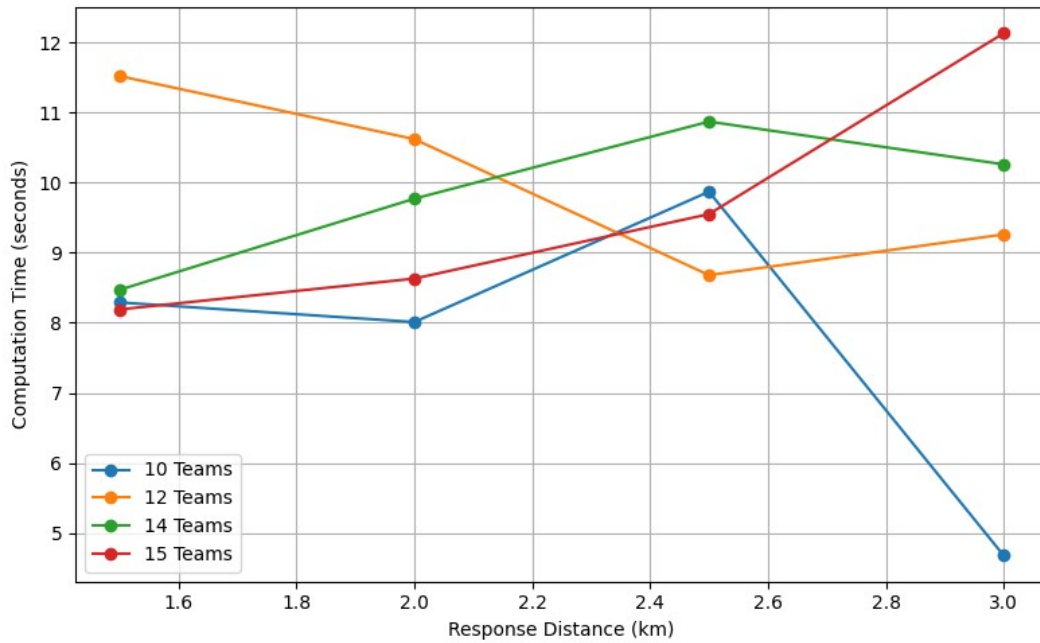


FIGURE 6. Computation Time Against Response Distance.

5. DISCUSSION

The results presented in Section 4 validate the CNM's capability to generate efficient and proficiency-aware patrol deployments. This section interprets these findings and discusses their implications.

5.1. Interpretation of Key Findings. The model successfully generated a deployment plan for 10 teams that achieved full coverage of all 225 crime-severity combinations while minimizing the total expected weighted response distance. The spatial allocation of teams (Figure 2) demonstrates the model’s ability to simultaneously optimize deployments for both geographic proximity and proficiency matching, a significant advancement over traditional location-allocation models. The sensitivity analysis yielded the following critical insights:

- i. **Robustness to Response Distance:** The invariance of the objective value and total travel distance to changes in response radius (Figures 3 and 5) indicates that the model identifies a fundamentally efficient spatial configuration. The expansion of the response radius simply increases the solution space without offering a more optimal configuration than the one already found, demonstrating the solution’s robustness. Figure 4 shows the corresponding decrease in travel distance percentage, which is an expected mathematical consequence of a fixed numerator (actual travel distance) and an increasing denominator (maximum possible travel distance).
- ii. **The 12-Team Anomaly:** The performance degradation observed with 12 teams (Table 4, Figures 3 and 5), where both the objective function and travel distance worsened compared to the 10-team configuration, is a notable finding. This anomaly suggests the existence of a suboptimal equilibrium. The addition of two teams disrupts the efficient spatial-proficiency clusters of the 10-team solution but is insufficient to form a superior configuration. This highlights a critical resource threshold and underscores that in a complex and constrained system, simply adding more resources does not necessarily improve performance and may even reduce it if deployment is not optimized as a whole.
- iii. **Computational Tractability:** Figure 6 shows that solution times remained consistently low (under 12 seconds) across all scenarios. This computational efficiency makes the CNM a viable tool for strategic planning, where decision-makers can rapidly evaluate multiple “what-if” deployment scenarios and resource allocations without delays.

Successful implementation of the model requires addressing several practical challenges through a phased approach. First, the trade-off between response radius and coverage must be carefully calibrated (with 1.5 km recommended) to balance response time and operational effectiveness. Second, collaboration with police agencies will be essential to obtain empirical proficiency data for model calibration. Finally, pilot testing should be conducted in medium-sized hotspots (25–100 hotspots) to refine parameters before scaling to larger metropolitan areas. Such pilots should also include officer training on how to interpret proficiency-aligned deployment strategies.

6. LIMITATIONS AND FUTURE WORK

Although the CNM offers a rigorous framework for optimizing patrol deployment, its design is constrained by its static formulation. The model relies on a

predefined set of scenarios and fixed input parameters, such as incident probabilities ($Q_i^{c,s}$), team proficiencies ($P_t^{c,s}$) and resource requirements ($L_i^{c,s}$), which are assumed to be known in advance. This limits its ability to adapt to the dynamic and unpredictable nature of crime in real-world settings. For example, the occurrence of a major incident could rapidly shift crime patterns and resource needs, leaving a pre-computed deployment plan suboptimal. In addition, the current structure does not integrate real-time data feeds, such as the operational status of officers already engaged in responses or prevailing traffic conditions, both of which are crucial for effective dispatch and rapid incident response.

A key avenue for future research lies in extending the CNM into a dynamic, real-time decision support system. This could be realized through a two-layer architecture: a predictive layer, which integrates crime forecasting models to continuously update incident probabilities ($Q_i^{c,s}$), and a reactive layer, which embeds the CNM within a rolling-horizon framework. By re-optimizing at regular intervals or in response to major events, the system would adapt to evolving conditions while utilizing previous solutions to preserve computational efficiency. Such an integrated system would connect long-term strategic planning with day-to-day operational decisions, using the CNMs proficiency-aware optimization to enable adaptive responses in dynamic urban contexts. Future research should also incorporate validation with real-world data to assess the models effectiveness under operational conditions.

7. CONCLUSION

This research has developed and validated the Crime Neutralization Model (CNM), an advanced optimization framework that significantly enhances police patrol deployment strategies for rapid incident response. The model's key contribution lies in its simultaneous consideration of three critical real-world factors: (1) varying patrol team proficiencies, (2) crime-type-specific resource needs and (3) scenario-dependent severity levels, elements frequently oversimplified in existing literature.

Our computational experiments demonstrate that the CNM achieves several operational advantages: Deployment plans remain optimal across expanding response distances, indicating the model identifies fundamentally efficient spatial configurations; the framework naturally limits team deployment to necessary locations, with 14 of 15 available teams being optimally utilized in maximum-resource scenarios; Consistent computation times under 12 seconds make the model practical for real-time decision support.

The sensitivity analysis revealed an important threshold effect. While 10 and 14-15 team configurations show expected performance improvements, the 12-team case exhibits paradoxical efficiency drops. This suggests the existence of critical resource thresholds in urban patrol deployment that warrant further investigation.

8. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the operational insights provided by the Kaduna State Police Command, whose practical expertise informed key model assumptions and constraints.

GLOSSARY OF SYMBOLS

For clarity and ease of reference, all mathematical symbols, parameters, and decision variables used in this study are summarized in the glossary below. This table ensures consistency in notation and facilitates a clear understanding of the CNM formulation and its associated components.

TABLE 5. Glossary of Symbols.

Symbol	Meaning	Units
K	Total number of patrol teams available for deployment	–
$d_{i,j}$	Shortest (Euclidean) distance between patrol location j and hotspot i	km
r	Maximum response distance	km
$\lambda_{i,j}$	Covering parameter: 1 if $d_{i,j} \leq r$, else 0	–
$L_i^{c,s}$	Level of police resources required at incident location i for crime category c under scenario s .	–
$Q_i^{c,s}$	Probability of crime category c under scenario s at location i	–
$P_t^{c,s}$	Proficiency of patrol team t in handling crime c under scenario s .	–
x_j	Binary: 1 if location j is assigned a patrol team; 0 otherwise	–
$y_{i,j}^{c,s}$	Binary: 1 if incident at location i is to responded to from location j for crime c under scenario s ; 0 otherwise	–
$z_{j,t}$	Binary: 1 if team t is assigned to location j ; 0 otherwise	–

Authors Contributions. K.M. Koko carried out the conceptualization, methodology, model formulation, software implementation, analysis and manuscript preparation. P. Ayuba, P. Anthony and S. Dari provided supervision, critical guidance and manuscript review.

Authors' Conflicts of interest. The authors declare no conflict of interest.

Funding Statement. No external funding was received specifically for the preparation or publication of this manuscript.

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