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WORK PERFORMED BY CLOSED AND CLOPEN M-TOPOLOGICAL FULL TRANSFORMATION SEMIGROUP SPACES M_{CT_n} AND $Clp(M_{CT_n})$

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ABSTRACT. Let α and β be two elements of an m-topological transformation semigroup space. In this paper, we introduce and derive explicit formulas for the work performed by elements in the full transformation semigroup M_{T_n}, the closed m-topological semigroup M_{CT_n}, and its clopen counterpart $Clp(M_{CT_n})$, where the displacement of a point x under a transformation α is given by $d(x, \alpha x) = |x - \alpha x|$. We further establish explicit formulas for the average work and the power within these semigroup spaces. To ensure that the system stabilizes to integer values, we incorporate the floor function $\lfloor x \rfloor = \{n \mid n \in \mathbb{Z}^+,\, n \leq x < n+1\}$. Numerical evaluations confirm the validity of the derived formulas and reveal consistent growth patterns, thereby highlighting new combinatorial properties of m-topological transformation semigroup spaces.

1. INTRODUCTION

Let $X_n = \{1, 2, 3, \ldots, n\}$. A transformation $\alpha : \operatorname{Dom} \alpha \subseteq X_n \to \operatorname{Im} \alpha \subset X_n$ is said to be *full* if its domain satisfies $\operatorname{Dom} \alpha = X_n$; otherwise, it is classified as *strictly partial* (see Umar [6]). The study of work performed by transformation semigroups was recently initiated by East and McNamara [2], inspired by a presentation delivered by Lavers at a semigroup special interest meeting in Sydney in 2004. Kehinde et al. [4] introduced the notion of *power* in transformation semigroups and provided graphical illustrations of their numerical results. In this review, we further examine the works of Daly and Vojte [1], as well as Laradji and Umar [5]. More recently, Francis, Adeniji, and Mogbonju [3], [8] introduced the notion of the m-topological transformation semigroup space, characterizing it as a family of transformation semigroups that fulfill the axioms governing topological spaces. Their study concentrated on a special class referred to as *Regular spaces*, denoted by M_{ψ_n}, where they analyzed the corresponding notions of work done and power. Francis *et al.*[9], Roots of Tropical Polynomials in Clopen and

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Non-Clopen Discrete m-topological transformation semigroups.
The aim of this paper is to explore the work performed by m-topological transformation semigroups, with particular emphasis on the quantification and assessment of total work, average work, and power within the framework of m-topological full transformation semigroup spaces, denoted M_{T_n}. Furthermore, this study extends to examine the work associated with a subclass termed the closed m-topological full transformation semigroup spaces, represented by M_{CT_n}, along with the clopen m-topological transformation spaces, symbolized as $Clp(M_{CT_n})$. The principal objective is to model elements of M_δ as collections of uniformly distributed points. We propose that the work executed by a transformation, when mapping an element x from the domain of a fixed set n to its image αx in the codomain, is captured by the metric distance $|x - \alpha x|$. Consequently, the cumulative work is computed as the aggregate of these distances as x varies across the domain of n. Specifically, the displacement incurred by x is expressed as $d(x, \alpha x) = |x - \alpha x|$. Accordingly, the overall work done on m_δ is obtained by summing these distances as α ranges over the domain corresponding to n. We denote the total work executed by the m-topological transformation semigroup by $w(m_\delta)$. This encompasses the total work $w(m_\delta)$, the mean (or average) work indicated by $\bar{w}(m_\delta)$, and the corresponding power, denoted $P_t(m_\delta)$.

Definition 1.1. [2] Let $\delta \in M_\delta$ and $x \in X_n$. We define the work of δ at x by

$$w(\delta) = \begin{cases} |x - \alpha(x)|, & \text{if } x \in \mathrm{Dom}(\alpha), \\ 0, & \text{otherwise.} \end{cases}$$

The *total work* of δ is defined as

$$w(M_\delta) = \sum_{\alpha \in M_\delta} \sum_{x \in X_n} w_x(M_\delta) = \sum_{\alpha \in M_\delta} \sum_{x \in Dom(\alpha)} |x - \alpha(x)|$$

The *average work* of δ is

$$\bar{w}(M_\delta) = \sum_{x \in X_n} \frac{w(M_\delta)}{|M_\delta|}.$$

Finally, the *power* of the transformations is defined by

$$P_t(M_\delta) = \sum_{x \in X_n} \frac{w_t(M_\delta)}{t}, \qquad \forall\, t \geq 1.$$

Lemma 1.2. *Let $\delta \subseteq M_\delta$. Then*

$$\delta = \sum_{\alpha(x)} |x - \alpha(x)|\, \Delta_{x,\alpha(x)}(M_\delta),$$

where $\Delta_{x,\alpha(x)}(M_\delta)$ denotes the set of all elements of M_δ that map x to $\alpha(x)$.

For $x, \alpha(x) \in X_n$, we set

$$\Delta_{x,\alpha(x)}(M_\delta) = \left|\Delta_{x,\alpha(x)}(M_\delta)\right|,$$

and for every $\alpha \in \Delta_{x,\alpha(x)}(M_\delta)$ we write

$$w_x(\alpha) = |x - \alpha(x)|.$$

Example 1.3. The elements of M_{CT_n} and $Clp(M_{CT_n})$ in the smallest case are as follows.

When $n = 1$:

$$M_{CT_1} = \left\{ \begin{pmatrix} 1 \\ \emptyset \end{pmatrix} \right\}, \qquad Clp(M_{CT_1}) = \varnothing.$$

When $n = 2$

$$|M_{CT_2}| = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ \emptyset & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 1 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ \emptyset & \emptyset \end{pmatrix} \right\} \tag{1.1}$$

$$|Clp(M_{CT_2})| = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \right\} \tag{1.2}$$

When $n = 3$

$$|M_{CT_3}| = \left\{ \begin{matrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & \emptyset & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & \emptyset & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & \emptyset \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 1 & \emptyset & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & \emptyset & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & \emptyset & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 2 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 2 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & 1 & \emptyset \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & 2 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ \emptyset & \emptyset & \emptyset \end{pmatrix} \end{matrix} \right\} \tag{1.3}$$

From equation (1.3) we have:

$$|Clp(M_{CT_3})| = \left\{ \begin{matrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \end{matrix} \right\} \tag{1.4}$$

Example 1.4. Consider arbitrary elements of M_{T_3} on $n = 3$, that is

$$M_{T_3} = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \right\} \tag{1.5}$$

Where

$$\alpha_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \end{pmatrix}, \alpha_4 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\alpha_1 \cup \alpha_2 = \alpha_3 \in M_{T_n}$$

$$\alpha_1 \cap \alpha_2 = \alpha_4 \in M_{T_3}$$

Similarly;

$$\alpha_1 \cup \alpha_3 = \alpha_3 \in M_{T_3}$$
$$\alpha_1 \cap \alpha_3 = \alpha_1 \in M_{T_3}.$$

M_{T_3} satisfies the properties of m-topological transformation semigroups.

Example 1.5. Consider the elements of M_{CT_2} in equation (1.1). To stabilize the average work done and power within integer values, we introduce the floor function, denoted as $\lfloor x \rfloor$, defined as $\{n \mid n \in \mathbb{Z}^+, n \leq x < n+1\}$. The work done, average work done, and power are given as follows:

$$w(M_{CT_2}) = \sum_{i \in n} w(M_{CT_2}) = 8,$$

The average work done by M_{T_2}

$$\bar{w}(M_{CT_2}) = \sum_{i \in n} \frac{w(M_{CT_2})}{|(M_{T_2})|} = \frac{8}{4} = 2.$$

The power of M_{T_2} for $t = 5$ is obtained as

$$P_5(M_{CT_2}) = \sum_{i \in n} \frac{w_5(m_{CT_2})}{5} = \frac{8}{5} = 1.$$

2. Main Result

Lemma 2.1. *Let $T_n \subseteq M_{T_n}$. Then*

$$M_{T_n} = n^n$$

Lemma 2.2. *For $n \geq 1$,*

$$\sum_{i=1}^{n} \sum_{j=1}^{n} |i-j| \;=\; \frac{n(n^2-1)}{3}.$$

Proof. If $i = j$, then $|i-j| = 0$, so only pairs with $i \neq j$ contribute. Since $|i-j| = |j-i|$, we may restrict to pairs with $i < j$ and double the result:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} |i-j| = 2 \sum_{1 \leq i < j \leq n} (j-i).$$

Fix $k = j - i$ with $1 \leq k \leq n-1$. The pairs (i,j) with $j - i = k$ are

$$(1, 1+k), (2, 2+k), \ldots, (n-k, n),$$

so there are exactly $n-k$ such pairs, each contributing k. Hence

$$\sum_{1 \leq i < j \leq n} (j-i) = \sum_{k=1}^{n-1} k(n-k).$$

Now compute:

$$\sum_{k=1}^{n-1} k(n-k) = n\sum_{k=1}^{n-1} k - \sum_{k=1}^{n-1} k^2.$$

Using the formulas

$$\sum_{k=1}^{n-1} k = \frac{(n-1)n}{2}, \qquad \sum_{k=1}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6},$$

we obtain

$$\sum_{k=1}^{n-1} k(n-k) = \frac{(n-1)n}{6}\big(3n-(2n-1)\big) = \frac{(n-1)n(n+1)}{6}.$$

Therefore,

$$\sum_{i=1}^{n}\sum_{j=1}^{n} |i-j| = 2\cdot\frac{(n-1)n(n+1)}{6} = \frac{n(n^2-1)}{3},$$

which proves the claim. □

Lemma 2.3. *Let $T_n \subseteq M_{T_n}$. Then*

$$w(M_{T_n}) = \frac{n^n(n^2-1)}{3}. \tag{2.1}$$

Proof. The number of transformations that map a fixed element $x \in X_n$ to a fixed value $y \in X_n$ is n^{n-1}, since the remaining $n-1$ elements of X_n may be mapped arbitrarily to any of the n values. Consequently, each ordered pair $(x, \alpha x) = (i, j)$ contributes exactly n^{n-1} occurrences in the summation that defines the total work.

Therefore

$$w(M_{T_n}) = \sum_{i=1}^{n}\sum_{j=1}^{n} |i-j|\; n^{n-1} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} |i-j|\right) n^{n-1}.$$

We now compute $S_n := \sum_{i=1}^{n}\sum_{j=1}^{n} |i-j|$. Note that

$$S_n = 2\sum_{1\le i<j\le n} (j-i) = 2\sum_{k=1}^{n-1}\sum_{i=1}^{n-k} k = 2\sum_{k=1}^{n-1} k(n-k).$$

Evaluate the last sum:

$$\sum_{k=1}^{n-1} k(n-k) = n\sum_{k=1}^{n-1} k - \sum_{k=1}^{n-1} k^2 = n\cdot\frac{(n-1)n}{2} - \frac{(n-1)n(2n-1)}{6} = \frac{n(n-1)(n+1)}{6}.$$

Hence

$$S_n = 2\cdot\frac{n(n-1)(n+1)}{6} = \frac{n(n^2-1)}{3}.$$

Substituting back gives

$$w(M_{T_n}) = \frac{n(n^2-1)}{3}\, n^{n-1} = \frac{n^n(n^2-1)}{3},$$

as required. □

Lemma 2.4. *Let $T_n \subseteq M_{T_n}$. Then*

$$\bar{w}(M_{T_n}) = \frac{n^2-1}{3}, \qquad P_t(M_{T_n}) = \frac{n^n(n^2-1)}{3t}.$$

Proof. By definition, the average work is

$$\bar{w}(M_{T_n}) = \frac{w(M_{T_n})}{|M_{T_n}|}.$$

From Lemma 2.3, we have

$$w(M_{T_n}) = \frac{n^n(n^2-1)}{3}, \qquad |M_{T_n}| = n^n.$$

Therefore,

$$\bar{w}(M_{T_n}) = \frac{\frac{n^n(n^2-1)}{3}}{n^n} = \frac{n^2-1}{3}.$$

The corresponding power is defined as work per unit time,

$$P_t(M_{T_n}) = \frac{w(M_{T_n})}{t} = \frac{n^n(n^2-1)/3}{t} = \frac{n^n(n^2-1)}{3t}.$$

□

Theorem 2.5. *Let $CT_n \subseteq M_{T_n}$ Then*

$$w(M_{CT_n}) \;=\; \frac{n^n(n^2+2)}{3}.$$

Proof. Write $d(i,j) = |i-j|$. Each ordered pair (i,j) contributes exactly $n^{\,n-1}$ occurrences to the total-work sum. Therefore

$$w(M_{CT_n}) = \sum_{i=1}^{n}\sum_{j=1}^{n} d(i,j)\, n^{\,n-1} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} |i-j|\right) n^{\,n-1}.$$

For $k \geq 1$ there are $2(n-k)$ ordered pairs (i,j) with $|i-j| = k$, while the diagonal pairs contribute 0. Hence

$$\sum_{i=1}^{n}\sum_{j=1}^{n} |i-j| = \sum_{k=1}^{n-1} k \cdot 2(n-k) = 2\sum_{k=1}^{n-1} k(n-k).$$

Evaluate the finite sum :

$$\sum_{k=1}^{n} k(n-k) = n\sum_{k=1}^{n-1} k - \sum_{k=1}^{n-1} k^2 = n\cdot\frac{(n-1)n}{2} - \frac{(n-1)n(2n-1)}{6} = \frac{n(n-1)(n+1)}{6}.$$

Substituting gives

$$w(M_{CT_n}) = 2n^{\,n-1}\cdot\frac{n(n-1)(n+1)}{6} = n^{\,n-1}\cdot\frac{n(n^2-1)}{3} = n^n\cdot\frac{n^2+2}{3},$$

which is the closed form. □

Theorem 2.6. *Let $CT_n \subset M_{T_n}$. Then*

$$\bar{w}(M_{CT_n}) = \left\lfloor \frac{n^2+2}{3} \right\rfloor, \qquad P_t(M_{CT_n}) = \left\lfloor \frac{n^2+2}{3t} \right\rfloor.$$

Proof. From Theorem (2.5) we have

$$w(M_{CT_n}) = n^n\left(1 + \frac{2}{n}\sum_{k=1}^{n} k(n-k)\right).$$

Since

$$\sum_{k=1}^{n} k(n-k) = \frac{n(n-1)(n+1)}{6},$$

one obtains the simplified total-work formula

$$w(M_{CT_n}) = n^n\left(1 + \frac{2}{n}\cdot\frac{n(n-1)(n+1)}{6}\right) = n^{\,n-1}\cdot\frac{n^2+2}{3}.$$

Therefore the average work (total work divided by $|M_{CT_n}| = n^n$) is

$$\bar{w}(M_{CT_n}) = \left\lfloor\frac{w(M_{CT_n})}{n^n}\right\rfloor = \left\lfloor\frac{n^2+2}{3}\right\rfloor.$$

Similarly, dividing the total work by $t\,n^{\,n-1}$ gives the time t of the work:

$$P_t(M_{CT_n}) = \left\lfloor\frac{w(M_{CT_n})}{t\,n^{\,n-1}}\right\rfloor = \left\lfloor\frac{n^2+2}{3t}\right\rfloor.$$

□

Theorem 2.7. *Let* $\mathrm{Clp}(M_{CT_n}) \subseteq M_{T_n}$ *Then*

$$\mathrm{Clp}(M_{CT_n} = \frac{n(2n-1)(n-1)^{\,n}}{6}.$$

Proof. By definition,

$$w(\mathrm{Clp}(M_{CT_n})) = \sum_{i=1}^{n}\sum_{j=1}^{n}|i-j|\cdot(n-1)^{\,n-1}.$$

For each $k \geq 1$ there are $2(n-k)$ pairs with $|i-j| = k$. Hence

$$w(\mathrm{Clp}(M_{CT_n})) = (n-1)^{\,n-1}\sum_{k=1}^{n-1} 2k(n-k).$$

We have

$$\sum_{k=1}^{n}(2k-1)(n-k) = 2\sum_{k=1}^{n} k(n-k) - \sum_{k=1}^{n}(n-k).$$

Evaluating gives

$$\sum_{k=1}^{n}(2k-1)(n-k) = \frac{n(2n-1)(n-1)}{6}.$$

Therefore

$$w(\mathrm{Clp}(M_{CT_n})) = (n-1)^{\,n-1}\cdot\frac{n(2n-1)(n-1)}{6} = \frac{n(2n-1)(n-1)^{\,n}}{6}.$$

□

Theorem 2.8. *Let $X_n = \{1, \ldots, n\}$. Assume* $\mathrm{Clp}(M_{CT_n}) \subseteq M_{T_n}$*, that for every ordered pair $(i, j) \in X_n \times X_n$ the multiplicity equals $(n-1)^{n-1}$, and that* $|\mathrm{Clp}(M_{CT_n})| = (n-1)^n$*. Set*

$$S := \sum_{k=1}^{n} (2k-1)(n-k).$$

Then

$$\bar{w}\big(\mathrm{Clp}(M_{CT_n})\big) = \frac{S}{n-1}, \qquad P_t\big(\mathrm{Clp}(M_{CT_n})\big) = \frac{S}{(n-1)\,t}.$$

Moreover, S has the closed form

$$S = \frac{n(2n-1)(n-1)}{6},$$

so equivalently

$$\bar{w}\big(\mathrm{Clp}(M_{CT_n})\big) = \frac{n(2n-1)}{6}, \qquad P_t\big(\mathrm{Clp}(M_{CT_n})\big) = \frac{n(2n-1)}{6t}.$$

Proof. By the multiplicity assumption,

$$w\big(\mathrm{Clp}(M_{CT_n})\big) = (n-1)^{\,n-1} \sum_{i=1}^{n} \sum_{j=1}^{n} |i-j| = (n-1)^{\,n-1}\, S.$$

Since $|\mathrm{Clp}(M_{CT_n})| = (n-1)^n$, the average work is

$$\bar{w}\big(\mathrm{Clp}(M_{CT_n})\big) = \frac{w(\mathrm{Clp}(M_{CT_n}))}{|\mathrm{Clp}(M_{CT_n})|} = \frac{(n-1)^{\,n-1}\, S}{(n-1)^n} = \frac{S}{n-1}.$$

The power (average work per unit time) is $\bar{w}/t$, hence

$$P_t\big(\mathrm{Clp}(M_{CT_n})\big) = \frac{S}{(n-1)\,t}.$$

Finally, evaluate S:

$$S = \sum_{k=1}^{n} (2k-1)(n-k) = \frac{n(2n-1)(n-1)}{6},$$

and substituting this closed form yields the simplified formulas above. □

n	$\|M_{T_n}\|$	$\|M_{CT_n}\|$	$\|Clp(M_{CT_n})\|$
1	1	1	0
2	4	4	1
3	27	27	8
4	256	256	81
5	3125	3125	1024
6	46656	46656	15625
7	823543	823543	279936
8	16777216	16777216	5764801
9	387420489	387420489	134217728
10	10000000000	10000000000	3486784401

TABLE 1. Number of elements in m-Topological Full transformation, clopen transformation semigroup in Closed m-Topological Full transformation semigroup spaces

n	$\|w(M_{T_n})\|$	$\|\bar{w}(M_{T_n})\|$	$\|P_5 w(M_{T_n})\|$
1	0	0	0
2	4	1	0
3	72	2	14
4	1280	5	256
5	25000	8	5000
6	544320	11	108864
7	13176688	16	26355337
8	352321536	21	70464307
9	10331213040	26	2066242608
10	330000000000	33	66000000000

TABLE 2. Total work done, Average Work done and power on m-Topological Full transformation semigroup spaces

n	$\|w(M_{CT_n})\|$	$\|\bar{w}(M_{CT_n})\|$	$\|P_5 w(M_{CT_n})\|$
1	1	1	0
2	8	2	1
3	99	3	19
4	1536	6	307
5	20625	9	4125
6	590976	12	118195
7	14000231	17	2800046
8	369098752	22	73819750
9	10718633530	27	2143726706
10	340000000000	34	68000000000

TABLE 3. Total work done, Average Work done and power on Closed m-topological Full transformation semigroup spaces

n	$\lvert w(Clp(M_{CT_n}))\rvert$	$\bar{w}(Clp(M_{CT_n}))$	$\lvert P_5 w(Cl(M_{CT_n}))$
1	0	0	0
2	1	1	0
3	20	2	4
4	378	4	75
5	7680	7	1536
6	171875	11	34375
7	4245696	15	849139
8	115296020	20	23059204
9	3422552064	25	6845104128
10	110414839365	31	22082967873

TABLE 4. Total work done, Average Work done and power on clopen m-topological Full transformation semigroup spaces

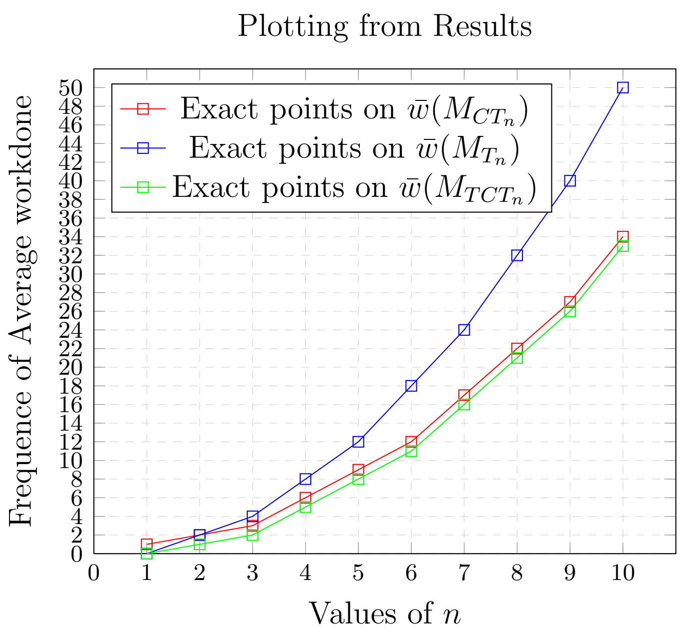

FIGURE 1. Average workdone on $\bar{w}(Clp(M_{CT_n}))$, $\bar{w}(M_{T_n})$ and $\bar{w}(M_{CT_n})$

We plot the graph of the relationship between the values of average work done from Table 2., Table 3., and Table 4. as shown in Figure 1.

3. CONCLUSION

It is essential to note that m-topological transformation semigroup spaces are not simply individual elements but rather structured sets of transformations that satisfy the properties of topological spaces. Our analysis demonstrates that the algebraic invariants derived for these spaces admit natural connections with integer sequences documented in [6]. Specifically, from Table 1., both $|M_{T_n}|$

and $|w(M_{CT_n})|$ correspond to `[A000312]`, while $|w(Cl(M_{CT_n}))|$ corresponds to `[A007778]`. Similarly, from Table 2, $|w(M_{CT_n})|$ aligns with `[A111868]`, and $|w(M_{T_n})|$ corresponds to `[A032765]`. At the time of writing, the sequences arising from the tabulated results in Tables 1. and 2. had not yet appeared in [6], thereby highlighting the novelty of our findings. These correspondences provide not only a new combinatorial interpretation of m-topological transformation semigroup spaces but also suggest possible directions for further research in algebraic combinatorics and topological semigroup theory.

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Authors Contribution

The first author carried out the main research work, including the formulation of the problem, derivation of results, and preparation of the manuscript. The second author, serving as the supervisor, provided overall guidance, critical insights, and valuable suggestions that shaped the direction of the research. Both authors discussed the results, reviewed the manuscript, and approved the final version for submission.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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