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VOLATILITY AND RISK ANALYSIS OF SELECTED COMPANIES IN NIGERIAN STOCK EXCHANGE

JOSEPHINE NNEAMAKA ONYEKA-UBAKA* AND KINGSLEY CHIMEZIE OHANMO[2]

ABSTRACT. The study introduces GARCH family models in modelling stock returns volatility on investor's decision-making and risk management in the Nigerian Stock Exchange market. Data (Dangote Cement PLC, Nigerian Flourmill, Guinness PLC, Nestle PLC and Unilever PLC) are sourced from ng.invest.com. The parameters of ARCH and GARCH models are estimated by maximum likelihood estimation method while the Lagrange Multiplier (LM) test is proposed testing heteroskedasticity. The results show that the data obtained within the sample period exhibit non-normality and no presence of autocorrelation in the squared standardized residuals. The FIGARCH model estimates the daily return series of Dangote Cement PLC, Nigerian Flourmill, Nestle PLC and Unilever PLC while the TGARCH model is more suitable for the Guinness PLC within the sampled period based on our diagnostic checks (Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC)). These findings are significant as they provide stakeholders with a deeper understanding of the patterns in the series, the leverage effect and make informed decisions on how to manage the associated risks. It is also important to note that the government's intervention in supporting struggling companies through policy creation is crucial, especially during periods of reduced returns and high inflation.

1. INTRODUCTION

Nigerian stock market, like stock markets worldwide, serves as a source of long-term financing for governmental development projects, private sector investments, and has acted as a catalyst during the banking system consolidation in the mid-2000s. Understanding the nature of the financial market return process, particularly the combination of drift and volatility, have been the focus of recent studies. Volatility can have detrimental effects on the smooth functioning of the financial system and overall economic performance. The volatility of stocks changes according to the relevance of the news as well as the degree to which the news surprise investors. Some financial experts and statisticians see the causes of volatility embedded in the arrival of new unanticipated information that altered returns on a stock [1]. Others claim that volatility are caused mainly by

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changes in trading volume, practices or patterns, which in turn are driven by factors such as modifications in macroeconomic policies, shift in investors' tolerance of risk and increase uncertainty [2]. According to [3], several factors contribute to the depreciation of stock prices in the Nigerian stock exchange market such as weak production base, import-dependent production structure, fragile export base, weak non-oil export earnings, expansionary monetary and fiscal policies, inadequate foreign capital inflow, excess demand for foreign exchange relative to supply, fluctuations in crude oil earnings, unguided trade liberalization policy, speculative activities, authorized dealers' sharp practices (round-tripping), over-reliance on an imperfect foreign exchange market, heavy debt burden, weak balance of payments position, and capital flight.

Stock market volatility analysis is important for decision-making in the case of capital and asset allocation. Policymakers use volatility estimates as an indicator of financial market vulnerability. Excessive volatility, however, undermines the reliability of stock prices as a signal for a firm's true intrinsic value. This challenges the concept of informational efficiency in markets. Having a clear understanding of volatility in the stock market is a veritable tool in assessing the cost of capital and making informed decisions about asset allocation. Activities in Nigeria, such as the recapitalization of the banking industry in July 2004 and the insurance industry in September 2005, have had a positive impact on the stock market. These activities led to an increase in the number of securities listed as well as heightened public awareness and confidence in the market. These increased trading activities may have influenced stock market volatility. However, in recent times, investors have expressed concerns about the declining stock prices in the Nigerian stock market. Also, the Nigerian stock market is a developing and inefficient one characterized by the time lag between information availability about a stock and its full reflection in the price of the stock, poor infrastructural facilities in the country which makes it virtually impossible for information to flow freely and speedily to actual and potential investors, activities of corporate insiders and insider abuses. Hence, an understanding of volatility and stock price forecasting in Nigerian stock market will be imperative as it helps in forecasting the path of its economy's growth and determines the efficiency of the stock market which will serve as an indicator of economic growth and development in Nigeria and in turn attract foreign portfolio investment.

Unlike linear time series where stocks are assumed to be uncorrelated but not necessarily identically independently distributed, nonlinear time series depicts that stocks are assumed to be identically independently distributed but there is a nonlinear function relating the observed time series $\{X_t\}_{t=0}^{\infty}$ and the underlying shocks $\{\varepsilon_t\}_{t=0}^{\infty}$. The effect of a violation of the assumption of heteroskedasticity is that the best linear unbiased estimation model is no longer consistent and as a result the regression output in terms of test statistics is longer reliable. This is because the variance which is in the heart of these statistics is no longer constant and will hence be misleading. Heteroskedasticity in the time series will take a graphically different appearance as the issue here is that the variance, or the volatility, will vary according to time. Some scholars like [4] and [5] have looked at the properties of volatility in the financial markets and there is wide recognition of the presence of heteroskedasticity in the distribution of returns. Mandelbroth [6] and [7] describe these phenomena as a clustering of volatility where a period of high volatility is likely to be followed by another period of high volatility and opposite. Engle and Ng [8] attribute the causes of volatility to the arrival of new, unanticipated information that alters expected returns on a stock. Changes in local or global economic environments, trading volume, trading practices or patterns can impact on information that is available to the market. Shiller [9] sees market volatility as a fundamental shift in investors' behaviour. Such behaviour is seen to be driven less by fundamental variables and more by sociological and psychological factors (behavioural finance model) as cultural changes and increasingly optimistic forecasts by analysts. Veronesi [10] and [11] see the behaviour as learning-induced phenomenon. They opined that the growth rate of the economy is unknown, and investors attempt to infer it from a variety of public signals. This inference process makes asset prices also depend on the investors' guesses about the dividend growth rate and thus induces higher return volatility. Roll [12] posits that volatility is affected by market micro-structure while [13] explained it by the liquidity provision process wherein when market makers infer the possibility of adverse selection, then they adjust their trading ranges which in turn increases the band of oscillation. Onyeka-Ubaka, *et al*. [14] show that the newly introduced generalized student-*t* distribution is the most general of all the useful distributions applied in the Bilinear GARCH (BL-GARCH) model parameter estimation.

Importantly, empirical studies relating to the probability of distributions of daily stock prices changes use dependent models where variance is conditional. Studies have focused on GARCH family processes where volatility varies over time and are persistent. In GARCH family models, variance changes over time as a function of past squared deviations from the mean and past variances. Researchers have confirmed the existence of relationship between volatility in consecutive periods, which could be utilized in forecasting future volatility in financial markets. These are accomplished through the application of modelling techniques such as autoregressive conditional heteroskedasticity (ARCH), generalized autoregressive conditional heteroskedasticity (GARCH), fractional integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) and threshold generalized autoregressive conditional heteroskedasticity (TGARCH). To adequately fit a GARCH family models, they are essential to conduct diagnostics checks alongside estimation. This approach is supported by previous research of [15]. In their research, they examined and compared estimates of different variations of GARCH models that incorporate breaks in relation to US dollar rates, with the break points determined exogenously. For our paper, we specifically focus on the selected GARCH family models to empirically capture the stylized facts present in the selected companies' stock prices using Gaussian, Student's *t*, and generalized error distribution (GED) to determine the most suitable volatility forecasting model with the appropriate error distribution.

2. MATERIALS AND METHODS

Engle [16] proposed the autoregressive conditional heteroskedastic (ARCH) model in which the conditional variance of a time series is a function of past shocks. The model provided a rigorous way of empirically investigating issues involving the volatility of economic variables in which the conditional variance σ_t^2 is given as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{2.1}$$

The ARCH model assumes that positive and negative shocks have the same effects on volatility. This is contrary to what is observed in practice where prices of financial assets respond differently to positive and negative shocks. The ARCH model is rather restrictive, for instance, α_1^2 of an ARCH(1) model must be in the interval [0, 1/3] if the series is to have a finite fourth moment. The ARCH model does not provide any new insight for understanding the sources of variation in financial time series data and sometimes, it over-parameterizes when the lag lengths elongate.

2.1 GARCH Model [17]

High frequency series such as stock returns are known with some stylized facts, common among which are volatility clustering, fat-tail and asymmetry. Mandelbrot [5] and [18] deduce that daily stock index returns are non-normal and tend to have leptokurtic and fat-tailed distribution. For this reason, [17] relaxed the traditional normality assumption to accommodate time varying volatility in high frequency data by assuming that such data follows student *t*-distribution. Furthermore, [19] established that a GARCH model with normally distributed errors could not be a sufficient model for explaining kurtosis and slowly decaying autocorrelations in return series. The generalized ARCH (GARCH) model introduces a conditional heteroskedasticity model that includes lags of the conditional variance as regressors in the model for the conditional variance (in addition to lags of the squared error terms $e_{t-1}^2, e_{t-2}^2, \cdots, e_{t-q}^2$). This model has only three parameters that allow an infinite number of squared errors to influence the current conditional variance (volatility). The general framework of this model, GARCH (p, q), is expressed by allowing the current conditional variance to depend on the first p past conditional variances as well as the q past squared innovations. That is,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{2.2}$$

where $\omega > 0;\ \alpha_i \geq 0\ and\ \beta_j \geq 0$, to ensure strictly positive conditional variance σ_t^2, p is the number of lagged σ^2 terms and q is the number of lagged ε^2 terms. The simple specification for GARCH(1,1) is:

Mean equation: $$r_t = \mu + \varepsilon_t \tag{2.3}$$

Variance equation: $$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2.4}$$

where $\omega > 0;\ \alpha_1 \geq 0\ and\ \beta_1 \geq 0$, r_t = return of the asset at time t, μ = average returns, ε_t = residual returns, defined as:

$$\varepsilon_t = \sigma_t z_t \tag{2.5}$$

where z_t is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1. The constraints $\alpha_1 \geq 0\ and\ \beta_1 \geq 0$, are needed to ensure σ_t^2 is strictly positive [4, 20]. In this model, the mean equation is written as a function of constant with an error term. Since σ_t^2 is the one-period ahead forecast variance based on past information, it is called the conditional variance. The conditional variance equation specified is a function of three terms: a constant term ω, news about volatility from the previous period measured as the lag of the squared residuals from the mean equation: ε_{t-1}^2 (the ARCH term); and last period forecast variance: σ_{t-1}^2 (the GARCH term). ARCH and GARCH models do not capture asymmetric effect since the lagged error terms are squared in the equations for the conditional variance, and therefore a positive error has the same impact on the conditional variance as a negative error.

2.2 EGARCH Models [21]

In the context of financial time series analysis, the asymmetry effect refers to the characteristic of time series on asset prices that an unexpected drop tends to increase volatility more than an unexpected increase of the same magnitude (or, that 'bad news' tends to increase volatility more than 'good news'). In an exponential GARCH (EGARCH) model, the natural logarithm of the condition variance is allowed to vary over time as a function of the lagged error terms (rather than lagged squared errors). The EGARCH (*p,q*) model for the conditional variance can be written as:

$$\ln(\sigma_t^2) = \omega + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^{q} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \tag{2.6}$$

The EGARCH model is asymmetric because the level $\frac{\varepsilon_{t-i}}{\sigma_{t-i}}$ is included with coefficient γ_i. Since the coefficient is typically negative, positive return shocks generate less volatility than negative return shocks assuming other factors remain unchanged. In order to capture asymmetric responses of the time-varying variance to shocks, this study employs EGARCH(1,1) model, which has the following specification:

Mean equation: $$r_t = \mu + \varepsilon_t \tag{2.7}$$

Variance equation: $$\ln(\sigma_t^2) = \omega + \beta_1 \ln(\sigma_{t-1}^2) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{2.8}$$

The EGARCH model which captures asymmetric properties between returns and volatility was proposed to address three major deficiencies of GARCH model. They are (i) parameter restrictions that ensure conditional variance positivity; (ii) non-sensitivity to asymmetric response of volatility to shock and (iii) difficulty in measuring persistence in a strongly stationary series. The log of the conditional variance in the EGARCH model signifies that the leverage effect is exponential and not quadratic. The specification of volatility in terms of its logarithmic transformation implies the non-restrictions on the parameters to guarantee the positivity of the variance [22], which is a key advantage of EGARCH model over the symmetric GARCH model. Malmsten and Terasvirta [23] argue that first order EGARCH model in normal errors is not sufficiently flexible enough for capturing kurtosis and autocorrelation in stock returns; however, they suggested how the standard GARCH model could be improved by replacing the normal error distribution with a more fat-tailed error distribution. This is possible because increasing the kurtosis of the error distribution will help standard GARCH model to capture the kurtosis and low autocorrelations in stock return series. Nelson [21] notes that a student-*t* distribution could imply infinite unconditional variance for the errors; hence, an error distribution with a more fat-tailed than normal will help to increase the kurtosis as well as reduce the autocorrelation of the squared observations. Nelson [21], therefore, assumes that EGARCH model is stationary if the innovation has a generalized error distribution (GED).

2.3 FIGARCH Model [24]

The fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) model was introduced by [24]. Under the FIGARCH model, the effect of the lagged squared innovations (unexpected return shocks) on the conditional variance decays with a slow hyperbolic rate. Therefore, the FIGARCH model can capture long-memory effects while still allowing the shocks to decay unlike the IGARCH model. To get a better understanding we take a look at the FIGARCH(p,d,q) model, in which p determines the number of autoregressive lags (GARCH effect) and q determines the number of moving average lags (ARCH effect). It is important to note that in the FIGARCH model, the coefficient of the ARCH effect is ϕ but not α as in the simple GARCH model. Nevertheless, it will still be interpreted in a similar way, namely as the ARCH parameter [25] and thus as the influence of the unexpected shocks. We will follow the general approach used in practice regarding the volatility modelling of stock returns and rely on first order models with only one lag, as they have proven to be a good representation of conditional variance processes [24]. Under the condition $0 < d < 1$, the process displays long memory for the conditional variance which will die out over time [25]. The closer the fractional differencing parameter d goes to one, the higher the memory of the FIGARCH model [26]. Furthermore, with d = 0 the FIGARCH collapses to a simple GARCH and with d = 1, it converges to the integrated GARCH (IGARCH) model. Modelling the return series as an ARMA (1,1) process:

$$r_t = \mu + \phi r_{t-1} + \eta_t + \gamma \eta_{t-1} \tag{2.9}$$

allows us to already incorporate an AR and a MA effect in the mean equation, with η_t being the unexpected return shock (error term). Given that η_t is a discrete time real-valued stochastic process, $\eta_t \equiv \varepsilon_t \sigma_t$ with $\varepsilon_t \sim \mathrm{N}(0,1)$, $\eta_t \sim \mathrm{N}(0, \sigma_t^2)$. We are able to model the conditional variance series as a FIGARCH(1,d,1) process, with σ_t representing the available information set at time $t-1$, as a time varying function, A FIGARCH(p,d,q) process can be displayed using the following equation:

$$\phi(L)[1 - \alpha(L)]\eta_t^2 = \omega + [1 - \beta(L)]v_t \tag{2.10}$$

In which L is defined as the backshift operator, which lags the coefficients:

$$\phi(L) = [1 - \alpha(L) - \beta(L)](1 - L)^{-1} \tag{2.11}$$

$$\begin{aligned} \alpha(L) &= \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_q L^q \\ \beta(L) &= \beta_1 L + \beta_2 L^2 + \cdots + \beta_p L^p \end{aligned} \tag{2.12}$$

The fractional differencing operator is given as:

$$(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)} L^k \tag{2.13}$$

In which Γ(z) defines the gamma function as:

$$\Gamma(z) = \int_{x=0}^{\infty} x^{z-1} e^{-x} dx \tag{2.14}$$

Rearranging equation (2.10) with $v_t = \eta_t^2 - \sigma_t^2$ yields the following expression for the FIGARCH (p,):

$$[1-\beta(L)]\sigma_t^2 = \omega + [1-\beta(L)-\phi(L)(1-L)^d]\eta_t^2 \tag{2.15}$$

The conditional variance of the stochastic process η_t is then given by:

$$\sigma_t^2 = \omega[1-\beta(L)] - 1 + \{1-[1-\beta(L)] - \phi(L)(1-L)^d\}\eta_t^2 \tag{2.16}$$

2.4 The Threshold GARCH (TGARCH) Model [27]

Another volatility model commonly used to handle leverage effects is the threshold GARCH (TGARCH) which allows the conditional standard deviation to depend on sign of lagged innovation. The version TGARCH(1,1) model specification of the conditional variance is given as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2.17}$$

where d_{t-1} is a dummy variable, that is,

$$d_{t-1} = \begin{cases} 1 & if\ \varepsilon_{t-1} < 0, bad\ news \\ 0 & if\ \varepsilon_{t-1} > 0, good\ news \end{cases} \tag{2.18}$$

The coefficient γ is known as the asymmetry or leverage term. When $\gamma = 0$, the model collapses to the standard GARCH forms, otherwise when the shock is positive (i.e. good news) the effect on volatility is σ_1. But when it is negative (i.e. bad news) the effect on volatility is $\alpha_1 + \gamma$. Hence, if γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks [28]. The specification does not show parameter restrictions to guarantee the positivity of the conditional variance. However, to ensure stationarity of the TGARCH model, the parameters of the model must be restricted and the choice of error distribution accounts for the stationarity. TGARCH model is closely related to GJR-GARCH model developed by [29]. Ding *et al.* [30] further generalized the standard deviation GARCH model initially proposed by [31] and [32] and called it Power GARCH (PGARCH). This model relates the conditional standard deviation raised to a power, *d* (positive exponent) to a function of the lagged conditional standard deviations and the lagged absolute innovations raised to the same power. This expression becomes a standard GARCH model when the positive exponent is set at two.

2.5 Estimation of the Model Parameters

The statistical techniques used for this paper include the non-normality distribution of fat tails; Augmented Dickey-Fuller test for stationarity and their model diagnostics. Regarding the return's estimation, as [33] pointed out "there are both theoretical and empirical reasons for preferring logarithmic returns. Theoretically, logarithmic returns are analytically more tractable when linking together sub-period returns to form returns over long time intervals. Empirically, logarithmic returns are more likely to be normally distributed and so, conform to the assumptions of the standard statistical techniques. This is the reason for using logarithmic returns in this study since one of the objectives was to test whether the daily returns were normally distributed or show signs of asymmetry (skewness). The computation formula for the daily returns is as follows:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{2.19}$$

where r_t is the return of the stock price in period t; P_t is the stock price in period *t* and, P_{t-1} is the stock price in period *t*-1. When estimating the parameters of ARCH or GARCH models, even though ordinary least squares (OLS) estimation is consistent, maximum likelihood estimator (MLE) is the most popular method, where parameters are chosen such that the probability of occurrence of data under its assumed density function is the maximum and produces an asymptotically normal and efficient parameter estimates. To test for this heteroskedasticity, the Lagrange Multiplier (LM) test proposed by [16] is applied. That is, an autoregressive moving average ARMA (1,1) model for the conditional mean in the returns series is employed as an initial regression and then, test the null hypothesis that there are no ARCH effects in the residual series. This implies that we obtain the residuals e_t from the ordinary least squares regression of the conditional mean equation. For an ARMA(1,1) model, the conditional mean equation is:

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{2.20}$$

In addition, the squared residuals, e_t^2 is regressed on a constant and q lags as in the equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_q e_{t-q}^2 \tag{2.21}$$

The null hypothesis, $\mathrm{H}_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$; states that there is no ARCH effect up to order q against the alternative, $\mathrm{H}_1 : \alpha_i > 0$; for at least one i = 1, 2, …, q. Finally, the test statistic for the joint significance of the q-lagged squared residuals is the number of observation times the R-squared ($\mathrm{T}R^2$) from the regression, where $\mathrm{T}R^2$ is evaluated against $\chi_{(q)}^2$ distribution.

We employ the Augmented Dickey–Fuller (ADF) test based on the following regression:

$$\Delta y_t = \varphi + \beta_t + \alpha y_{t-1} + \sum_{i=1}^{k} d_i \Delta y_{t-1} + u_t \tag{2.22}$$

where u_t is a white noise error term and $\Delta y_{t-1} = y_{t-1} - y_{t-2}, \Delta y_{t-2} = y_{t-2} - y_{t-3}$, etc. Equation (2.22) tests the null hypothesis of a unit root against a trend stationary alternative. The Philips-Perron (PP) test is equally conducted on the return series, which uses models like the Dickey-

Fuller tests but with Newey-West non-parametric correction for possible autocorrelation rather than the lagged variable method employed in the ADF test. The Philips-Perron test is computed from the equation below:

$$y_t = \delta_t + \gamma y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_p \Delta y_{t-p} + \mu_t \qquad (2.23)$$

where δ_t is the deterministic trend component, and maybe 0, φ or $\varphi + \beta_t$, γ is the coefficient of the of the lagged value.

To estimate the parameters of the underlying FIGARCH model, we again rely on the approach proposed by [24]. To obtain maximum likelihood estimates for the FIGARCH(p,d,q) model the following log likelihood function must be optimized:

$$logL(\theta; \eta_1, \eta_2, \dots, \eta_T) = -0.5Tlog(2\pi) - 0.5\sum_{t=1}^{T}[log(\sigma_t^2) + \eta_t^2 \sigma_t^{-2}] \qquad (2.24)$$

where $\theta' = (\omega, d, \beta_1, \dots, \beta_p, \phi_1, \dots, \phi_q) = (\omega, d, \beta_1, \phi_1)$ displays the starting values under which the log likelihood function is maximized. The global maximum can be achieved using the maximum Likelihood estimation. The second equality holds since we use a first order model with only one lag each. These parameters are namely $\omega, d, \beta_1, \phi_1$ respectively, a constant, the long-memory parameter, the GARCH effect and the ARCH effect. Using the global maximum obtained, the parameter of the appropriate model can be estimated through a search algorithm that tries several different coefficients before converging on the optimum values. To verify our obtained results, we manually compare the conditional volatility series from each subseries, with the sample volatility series as well as the conditional volatility series of the whole data set, to see whether the according parts resemble each other. In certain cases, we obtain implausible parameter estimates from starting values that result in log likelihood, which is an outlier and is only above the second largest log likelihood value by an extremely small margin. If this happens, we manually adjust the optimal starting parameters to yield more plausible results, namely by choosing the starting values which result in the next largest log likelihood value. After estimating the parameters of our models, we use statistical diagnostic checks to select the best model(s) to be used for forecasting.

3. RESULTS

We analyze the data on daily stock prices of selected companies from 27th March, 2012 to 30th December, 2022 using E-Views statistical packages. The descriptive (summary) statistics are presented on Table 3.1.

Table 3.1: Summary Statistics of Stock Returns

	Dangote PLC	Flour mill	Guinness PLC	Nestle	Unilever
Mean	0.000583	0.000203	-0.000112	0.000633	4.18E-05
Median	0.000000	0.000000	0.000000	0.000000	0.000000
Maximum	0.102500	0.132200	0.102400	0.102500	0.126200
Minimum	-0.100000	-0.124400	-0.100900	-0.10000	-0.104700
Std. Dev.	0.021001	0.030867	0.026964	0.021170	0.028806
Skewness	0.445508	0.057832	0.233610	0.290491	0.134751
Kurtosis	9.586513	5.782551	7.296037	9.714096	5.867516
Jarque-Bera	4677.141	782.7065	1960.016	4681.697	868.5850
Probability	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	1.482400	0.000000	-0.281200	1.565500	0.105100
Sum Sq. Dev.	1.120269	0.491400	1.830755	1.108274	2.084478
Observation	2541	2422	2519	2474	2513

Table 3.1 presents the descriptive statistics for the variables. For the period under examination, Dangote cement PLC exhibits a mean, median, maximum, and minimum of 0.000583, 0.000000, 0.102500, and -0.100000 respectively. The standard deviation and Jarque-Bera statistic value are 0.021001 and 4677.141 respectively, at a 5% level of significance. Similarly, Nigerian Flourmill PLC has a mean, median, maximum, and minimum of 0.000203, 0.000000, 0.132200, and -0.124400 respectively. The standard deviation and Jarque-Bera statistic value are 0.030867 and 782.7065 respectively, at a 5% level of significance. Guinness PLC has mean, median, maximum and minimum of -0.000112, 0.000000, 0.102400, and -0.100900 respectively for the time period examined. And has standard deviation and Jarque-Bera statistic value of 0.026964 and 1960.016 respectively, at 5% level of significance. Furthermore, Nestle PLC has mean, median, maximum and minimum of 0.000633, 0.000000, 0.1025000 and -0.10000 respectively for the time period examined. And has standard deviation and Jarque-Bera statistic value of 0.021170 and 4681.697 respectively at 5% level of significance. Unilever PLC has mean, median, maximum and minimum of 4.18E-05, 0.000000, 0.126200 and -0.104700 respectively for the time period examined. And has standard deviation and Jarque-Bera statistic value of 0.028806 and 868.5850 respectively at 5% level of significance.

From the summary statistics of the variables, the skewness, kurtosis and Jarque Bera statistics confirm non-normality for all the variables at 5% level of significance. Hence, Figures 3.1 to 3.5 below present the time series plots of the selected companies. The plots show that the series exhibits non-stationary behaviour which implies time-varying volatility in daily stock prices.

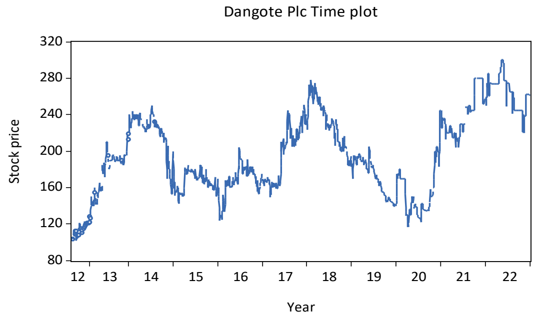


Figure 3.1: Dangote cement Plc Time Plot

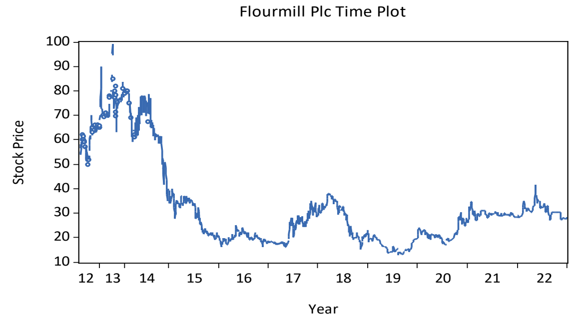


Figure 3.2: Nigerian Flourmill Plc Time Plot

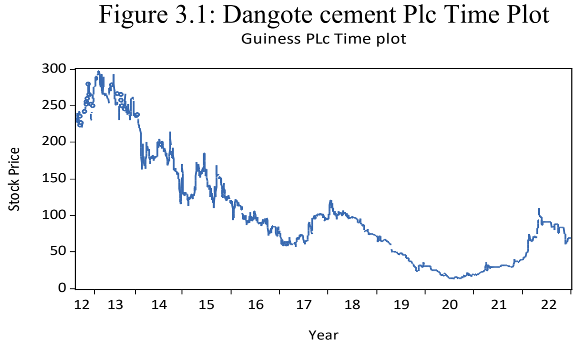


Figure 3.3: Guinness Plc Time Plot

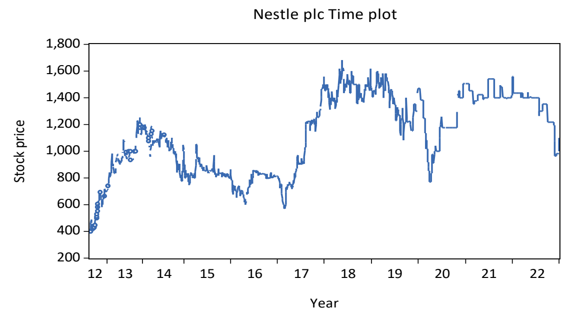


Figure 3.4: Nestle Plc Time Plot

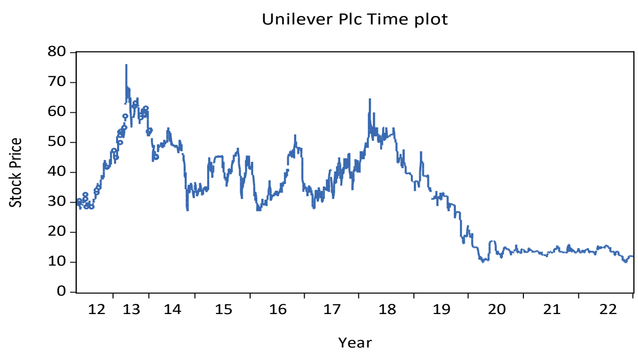


Figure 3.5: Unilever Time Plot

The results from the ADF test with a linear time trend are reported in Table 3.2. Using the ADF test with trend, the unit root cannot be rejected for five variables at 5% level of significance. Hence the results conform to the time series plots presented earlier

Table 3.2: Unit Root Test (ADF Test) for Stock Price

Variables	$\hat{k}$	Test Statistics	P-Value
Dangote Cement Plc.	0	-2.86248	0.1067
Flourmill Plc.	1	-2.86253	0.3616
Guinness Plc.	1	-2.86249	0.4294
Nestle Plc.	0	-2.86251	0.0643
Unilever Plc.	1	-2.86249	0.6356

Note: $\hat{k}$ *is the AIC lag term used to select the optimal lag, to make the residuals white noise*

Since all the p-values for the stock price are greater than 0.05 (5% level of significance); we do not have enough reasons to reject the unit root null hypothesis. Hence, the stock price for Dangote cement PLC, Flourmill PLC, Guinness PLC, Nestle PLC, and Unilever PLC are nonstationary. Hence, we take the first difference of the series and further test for stationarity of stock returns.

Table 3.3: Unit Root Test (ADF Test)

Variables	$\hat{k}$	Test Statistics	P-Value
Dangote Plc.	0	-2.86248	0.0001
Flourmill Plc.	0	-2.86253	0.0001
Guinness Plc.	4	-2.86249	0.0000
Nestle Plc.	2	-2.86251	0.0000
Unilever Plc.	4	-2.86249	0.0000

Note: $\hat{k}$ *is the AIC lag term used to select the optimal lag, to make the residuals white noise*

From Table 3.3, Since the absolute *t*-statistics values of all the series are greater than the absolute critical values 2.8625 or *p*-values less than 0.05 (5% level of significance), we have enough reasons to reject H_0 at first difference for Dangote cement PLC, Nigerian Flourmill PLC, Guinness PLC, Nestle PLC and Unilever PLC. Hence, the series obtained are stationary.

To test stationarity, the autocorrelation function (ACF) and partial autocorrelation function (PACF) are plotted on figures 3.6 - 3.10. The plots show that the ACF and PACF of the residuals are not autocorrelated. After the lag-0 correlation, the subsequent correlations drop quickly to zero and stay (mostly) between the limits of the significance level (dashed blue lines). Therefore, we can conclude that our data for the five companies meets the assumption of no autocorrelation. The ACF cuts off after lag one indicating a moving average of order one (MA(1)) while the PACF cuts off after lag one pointing to autoregressive model of order one (AR(1) for all the series. It is also observed that there are spikes after lag one indicating the autoregressive moving average (ARMA(1, 1)) model should be tried as well to aid in the lag length of our models during estimation.

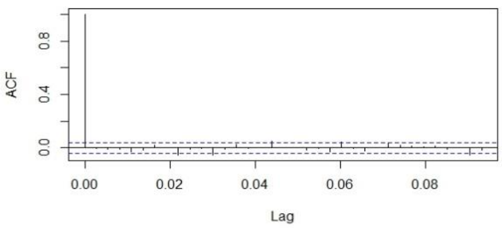


Figure 3.6a: ACF of Dangote cement PLC

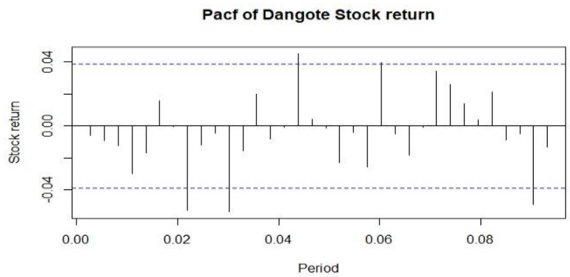


Figure 3.6b: PACF of Dangote cement PLC

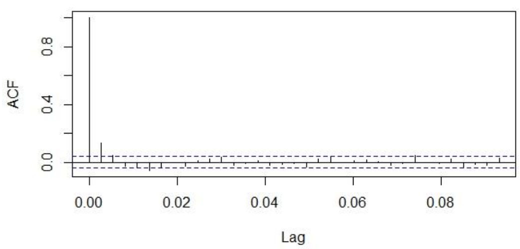


Figure 3.7a: ACF of Nigerian Flourmill PLC

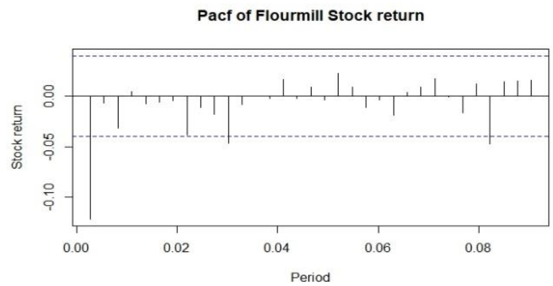


Figure 3.7b: PACF of Nigerian Flourmill PLC

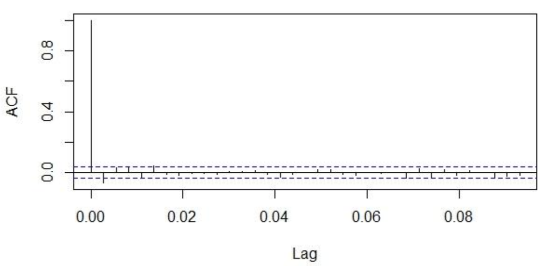


Figure 3.8a: ACF of Nestle PLC

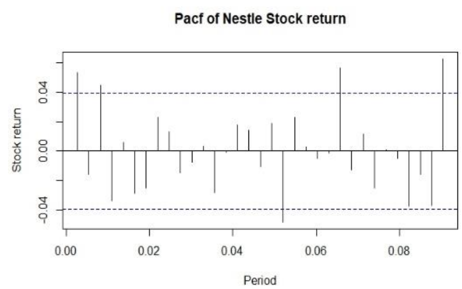


Figure 3.8b: PACF of Nestle PLC

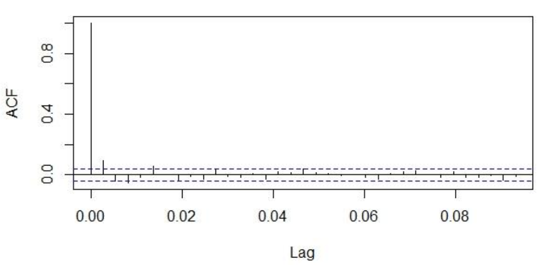


Figure 3.9a: ACF of Guinness PLC

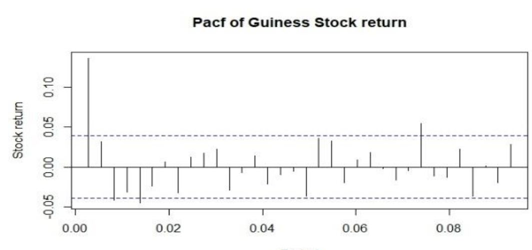
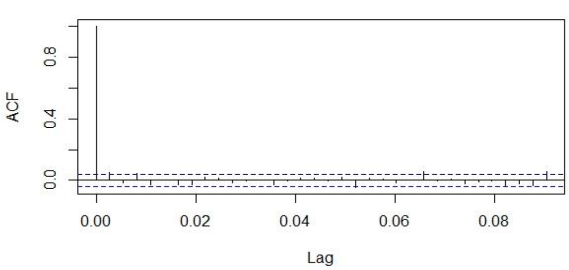


Figure 3.9b: PACF of Guinness PLC



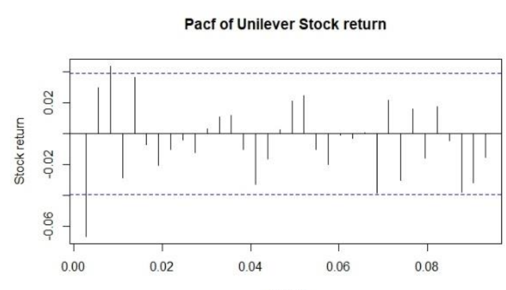


Figure 3.10a: ACF of Unilever PLC

Figure 3.10b: PACF of Unilever PLC

Before estimating the ARCH and GARCH models, we investigate the stock return series to identify its statistical properties and to see if it meets the pre-conditions for the ARCH and GARCH models, that is, clustering volatility and ARCH effect in the residuals. The series of daily stock returns for the five companies seem to randomly fluctuate around zero, meaning there is little autocorrelation. This is confirmed by a plot of the sample autocorrelation function and the partial autocorrelation function. The figures 3.11 - 3.15 show that return series oscillate around the mean value (mean reverting) and approximately constant variance. Volatility of stock returns are high for consecutive periods and low for another consecutive period, this feature of sustained periods of calmness and sustained periods of high volatility was first observed by Mandelbroth (1963) as volatility clustering, a stylized fact financial time series exhibit, a condition necessary for the application of GARCH family models.

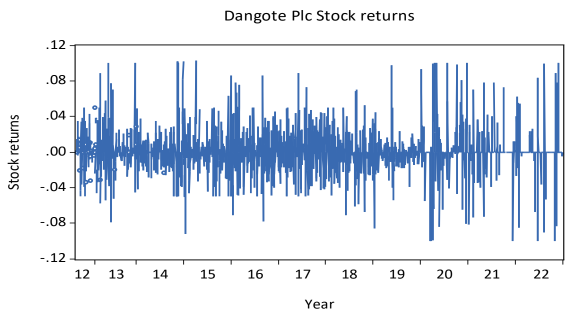


Figure 3.11: Plot of Dangote cement PLC Stock returns

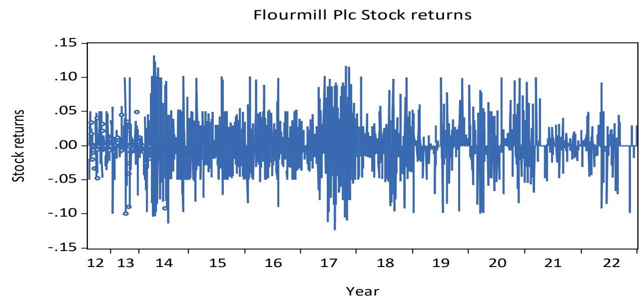


Figure 3.12: Plot of Nigerian Flourmill PLC Stock returns

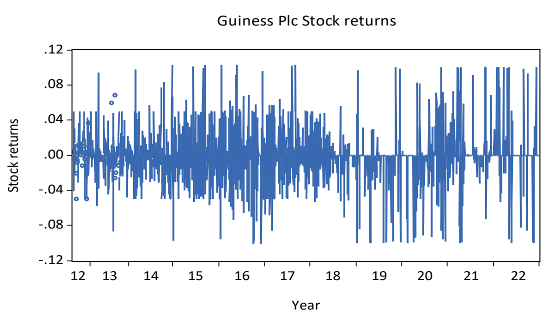


Figure 3.13: Plot of Guinness PLC Stock returns

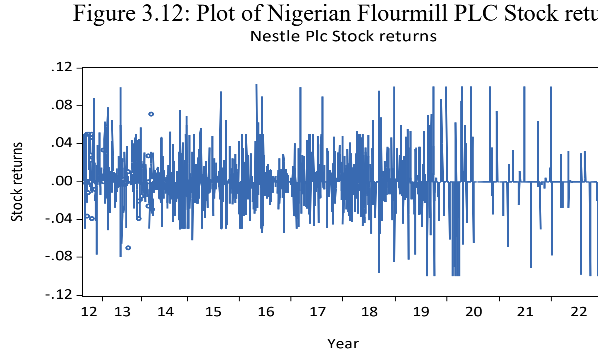


Figure 3.14: Plot of Nestle PLC Stock returns

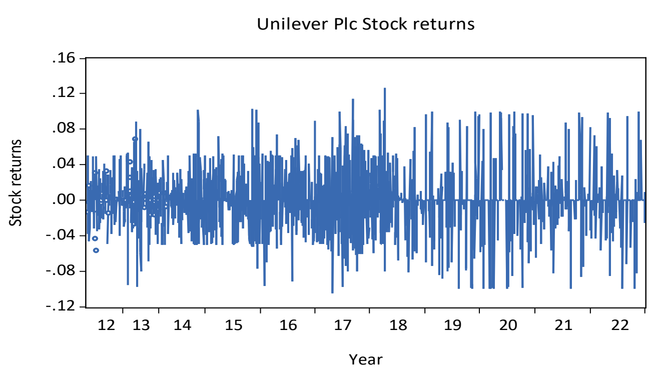


Figure 3.15: Plot of Unilever PLC Stock returns

The ARCH effect is concerned with a relationship within the heteroskedasticity, often termed serial correlation of the heteroskedasticity. It often becomes apparent when there is bunching in the variance or volatility of a particular variable, producing a pattern which is determined by some factor (see Figures 3.11 to 3.15). The stock returns are further subjected to Heteroskedasticity test presented on Table 3.4.

Table 3.4: Results of test for ARCH effect on daily Returns Data

Heteroskedasticity Test	Dangote	Flourmill	Guinness	Nestle	Unilever
F-statistic Prob.F	93.78251 (1,2537) 0.0000*	182.7059 (1,2418) 0.0000*	136.3554 (1,2515) 0.0000*	57.19592 (1,2470) 0.0000*	72.63143 (1,2509) 0.0000*
Obs*R-squared Prob. Chi-square(1)	90.51063 0.0000*	170.0109 0.0000*	129.4457 0.0000*	55.94671 0.0000*	70.64429 0.0000*

Note: The asterisks are the P-values. The hypothesis is:

$\boldsymbol{H_0}$: *no ARCH effects vs.* $\boldsymbol{H_1}$: *ARCH (p) disturbance.*

Table 3.4 shows the results of the test for ARCH effect. ARCH-effect is present if the coefficient of the lagged value of residual squared (ε_{t-1}^2) is positive and if the estimate is statistically significant. From the results in table 3.4, the coefficient of (ε_{t-1}^2) are positive. Also, based on the t-test as well as F-test and Chi-square test, the estimate is significant at the 5% level. Therefore, the null hypothesis that there is no ARCH-effect is rejected. Having established that ARCH-effect is present, the parameters of the appropriate model were estimated through a search algorithm that tries several different coefficients before converging on the optimum values.

Table 3.5: Parameter Estimates of Selected Models for Daily Returns Data of five companies

	ω	α	β	γ
GARCH (1, 1) Dangote	1.32E-14 (1.30E-14, 1.016941)	0.299309 (0.022309, 13.41680)	0.591369 (0.003771, 156.8204)	
Flourmill	5.83E-05 (4.45E-06, 13.09370)	0.157194 (0.011344, 13.85712)	0.784845 (0.011793, 66.55206)	
Guinness	0.000203 (1.26E-05, 16.04500)	0.220118 (0.020223, 10.88439)	0.504311 (0.027093, 18.61404)	
Nestle	9.95E-05 (6.50E-06, 15.31144)	0.117022 (0.010710, 13.85712)	0.663920 (0.019731, 33.64923)	
Unilever	0.000133 (1.27E-05, 10.45564)	0.128804 (0.012826, 10.04264)	0.711782 (0.023329, 30.51001)	
EGARCH (1,1)				0.046772

Dangote	-2.614236 (0.149385, -17.4999)	0.333954 (0.018189, 18.36069)	0.690248 (0.018365, 37.58429)	(0.013177, 3.549486)
Flourmill	-0.897244 (0.054137, -16.5735)	0.1343194 (0.011344, 17.85542)	0.896487 (0.006775, 132.4323)	0.084845 (0.011796, 2.655206)
Guinness	-2.815584 (0.172059, -16.3641)	0.220132 (0.020234, 14.69245)	0.644770 (0.022429, 28.74699)	0.070204 (0.053717, 3.167308)
Nestle	-3.041865 (0.188568, -16.1314)	0.276426 (0.010743, 15.75597)	0.627597 (0.023492, 26.71510)	0.103483 (0.015469, 6.689808)
Unilever	-2.029926 (0.181625, -11.1765)	0.128843 (0.012823, 12.58177)	0.738185 (0.024057, 30.51034)	-0.002223 (0.013301, -0.155474)
TGARCH (1,1)				
Dangote	2.41434 (0.14955, -17.47394)	0.338964 (0.01839, 18.3689)	0.673100 (0.01315, 60.4731)	0.369200 (0.072352, 5.093946)
Flourmill	-2.51430 (0.12334, -15.4565)	0.245442 (0.01245, 17.66768)	0.668220 (0.015336, 43.57113)	2.3947 (0.025763, -8.6459)
Guinness	-3.45341 (0.34232, 16.34567)	0.333457 (0.01293, 17.95847)	0.672504 (0.041932, 10.29473)	-0.017722 (0.030862, -0.219166)
Nestle	2.61876 (0.23467, -15.4376)	0.234561 (0.01235, 17.68935)	0.523047 (0.01451, 13.9484)	0.0474051 (0.012638, 3.753931)
Unilever	-1.65673 (0.12334, -15.4565)	0.235448 (0.01879, 17.68978)	0.685040 (0.060928, 11.24339)	0.03692 (0.072518, 5.09391)
FIGARCH(1,1)				
Dangote	0.134319 (0.149385, -17.4994)	-0.333954 (0.018189, 18.36069)	0.614236 (0.011344, 17.85542)	-0.084845 (0.011796, 2.655206)
Flourmill	0.514430 (0.12334, -15.45650)	0.245440 (0.01245, 17.66768)	0.644770 (0.022429, 28.74699)	-0.071255 (0.017711, 3.963787)
Guinness	0.476558 (0.038015, 12.53594)	0.055093 (0.013537, 4.069656)	0.650044 (0.033248, 19.55111)	-0.000169 (1.52E-05, 11.10318)
Nestle	-2.029926 (0.181625, -11.1764)	0.23458 (0.01235, 17.68935)	0.31876 (0.021347, -15.4376)	0.128843 (0.012823, 12.58177)
Unilever	1.15673 (0.12334, -15.4565)	-0.23544 (0.01879, 17.68978)	0.606232 (0.023595, 55.36020)	0.177901 (0.013539, 13.13971)

Note: The bracketed is the Standard error and Z-statistic respectively. The Variance Equation of GARCH (1, 1) for example for the five Companies (Dangote PLC, Flourmills, Guinness, Nestle and Unilever are: $\alpha + \beta = 0.89$, $\alpha + \beta = 0.94$, $\alpha + \beta = 0.72$, $\alpha + \beta = 0.78$ and $\alpha + \beta = 0.84$, respectively.

In all the estimation, the results of the variance (volatility) equation is presented in Table 3.5. In GARCH (1, 1) model, the coefficients ω (constant), ARCH term (α) and GARCH term (β) are statistically significant at 5% level of significance for the Dangote cement PLC, Nigerian Flourmill

PLC, Guinness PLC, Nestle PLC and Unilever PLC return series. The significance of both α and β indicates that, lagged conditional variance and lagged squared residuals have an impact on the conditional variance. In other words, news about volatility (i.e., fluctuation) from previous periods has an explanatory power over current volatility. The sum of the two estimated ARCH and GARCH ($\alpha + \beta$) coefficients which is regarded as the persistence coefficient is less than one for all the selected companies' return series, which is required to have a mean reverting process. These values closer to 1 indicate that shocks to volatility are very high and will remain for a very long period. All the coefficients in EGARCH (1, 1) are statistically significant at 5% confidence level for the Dangote cement Plc., Nigerian Flourmill Plc., Guinness Plc., Nestle Plc and Unilever Plc return series. The estimates of the leverage effect (γ) are positive and significant at 5% confidence level for the returns. These results are inconsistent with the conventional situation where negative signs are assumed for leverage effect presence. It is therefore evident that in all our series, volatility can be said to be positively correlated with returns, i.e., falling returns are followed by lower volatility, indicating the nonexistence of leverage effects in the daily series during the study period. In TGARCH (1, 1) model, the estimates of the leverage effect (γ) are positive and significant at 5% confidence level for the returns except for Guinness which is negative and not significant at 5% confidence level for the returns. The positive γ (i.e. good news) indicate that the effect on volatility is σ_1 while the negative γ (i.e. bad news) indicates the effect on volatility is $\alpha_1 + \gamma$. Looking at FIGARCH (1, 1), all the coefficients are statistically significant at 5% confidence level for the Dangote cement Plc., Nigerian Flourmill Plc., Guinness Plc., Nestle Plc. and Unilever Plc return series. The estimates of the leverage effect (γ) are a mixture of negative and positive, and significant at 5% confidence level for the returns. It is therefore evident that in all our series (Dangote cement, Nigerian Flourmill and Guinness), bad news has a negative impact on the return's volatility than good news. That is, falling returns are followed by high volatility indicating the existence of leverage effects except for the Nestle and Unilever return series during the study period.

The diagnostic checks of the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) for the GARCH, EGARCH, FIGARCH and TGARCH models are summarized in Table 3.6. The results show that the FIGARCH model is preferred for evaluating daily return series for Dangote cement Plc., Flourmill Plc., Nestle Plc. and Unilever Plc. while the TGARCH modeling technique is preferred for the daily return series for Guinness Plc. during the study period. The criteria are to pick the least AIC and BIC values as the best fit model for our data.

Table 3.6: AIC and BIC Values for Selected Models

Companies/Models	ARCH(1)		GARCH(1,1)		EGARCH(1,1)		FIGARCH(1,1)		TGARCH(1,1)	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Dangote Plc.	-4.9970	-4.9811	-5.0161	-5.0004	-5.0032	-4.9872	-5.0214	-5.0054	-5.0177	-5.0017
Flourmill Plc.	-4.2385	-4.2042	-4.3513	-4.3171	-4.1321	-4.0979	-4.3659	-4.3317	-4.3511	-4.3169
Guinness Plc.	-4.5038	-4.5085	-4.5335	-4.5388	-4.5244	-4.5297	-4.5337	-4.5390	-4.5373	-4.5426
Nestle Plc.	-4.9346	-4.5163	-4.9461	-4.5283	-4.9531	-4.5353	-4.9661	-4.5483	-4.9546	-4.5368
Unilever Plc	-4.3112	-4.5024	-4.3426	-4.5338	-4.3345	-4.5257	-4.3478	-4.5398	-4.3420	-4.5332

Having selected the models, volatility forecasts for thirty days of the series based on the models are presented in Figures 3.16 - 3.20.

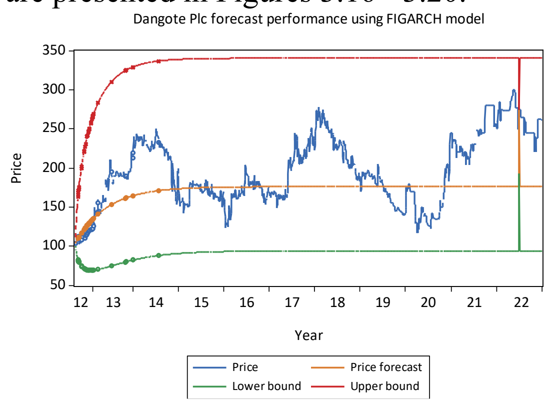


Figure 3.16: Dangote Forecast performance using FIGARCH Model

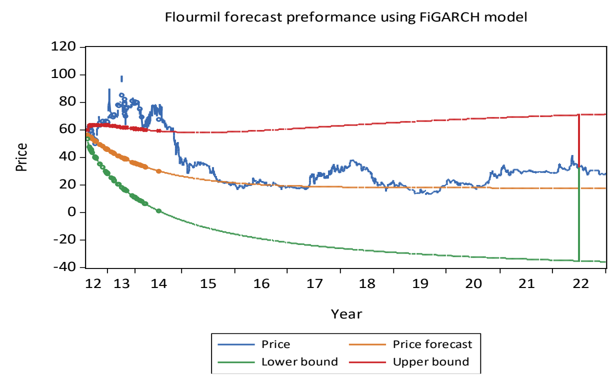


Figure 3.17: Flourmill Forecast performance using FIGARCH Model

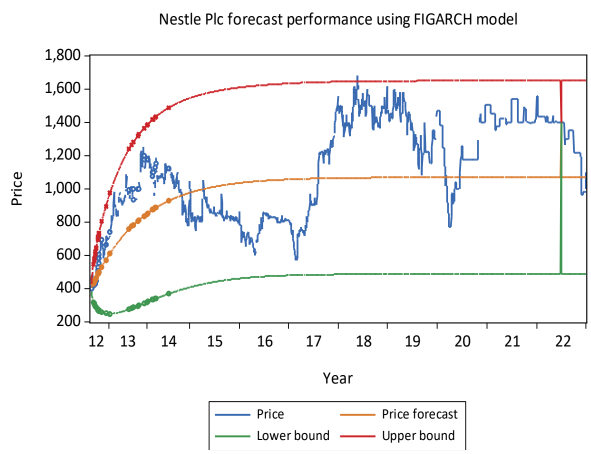


Figure 3.18: Nestle Forecast performance using FIGARCH Model

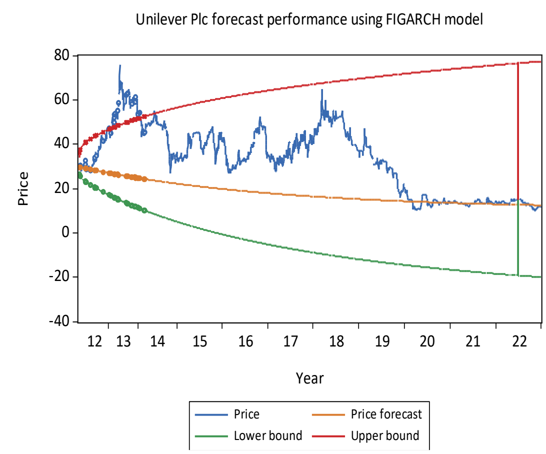


Figure 3.19: Unilever Forecast performance using FIGARCH Model

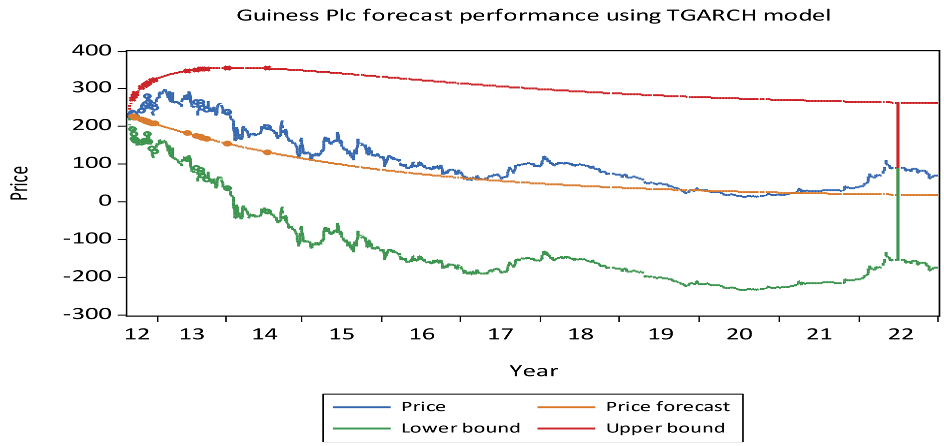


Figure 3.20: Guinness Forecast performance using TGARCH Model

4. DISCUSSION

The parameters of the selected GARCH family models are estimated by maximum likelihood estimation method while the Lagrange Multiplier (LM) test is proposed for testing heteroskedasticity. The sum of the two estimated ARCH and GARCH ($\alpha + \beta$) coefficients which is regarded as the persistence coefficient is less than one for all the selected companies' return series, which is required to have a mean reverting process. All the coefficients are statistically significant at 5% confidence level for the Dangote cement Plc., Nigerian Flourmill Plc., Guinness Plc., Nestle Plc. and Unilever Plc return series when FIGARCH (1, 1) model was employed.

The analysis also shows that the FIGARCH model that has been fitted seems appropriate for the data at a 1% confidence level because the autocorrelation function (AC), partial autocorrelation function (PAC), and Q-statistics show that there is no statistically significant trace of autocorrelation left in the squared standardized residual indicating that the mean equation and variance equation are adequately specified. The AIC and BIC metrics, which measure how effectively the model used for the analysis captures the empirical features in high frequency time series select the model(s) with minimal values. Modern applications of time series analysis using Bayesian analysis, convolutional neural networks, a non-linear generalization of autoregressive models, a natural blend of practical time series analysis, and machine learning may be employed for further comparison.

5. CONCLUSION

This paper has examined the best models for capturing more stylized facts (volatility clustering, leverage effects and leptokurtosis) and to provide a better in-sample fit and/or out-of-sample forecast. The in-sample period was from 27th March 2012 to 30th December 2022 and the out-of-sample was done one hundred and twenty-five days ahead. The discrepancy in the performance of the volatility models in both in-sample and out-sample forecast can result from the fact that the dynamics of the volatility may have changed during the long-time horizon of the data and the volatility of stock prices may have shifted over time. The dynamics of volatility are stationary and are expected to be steady, especially over a long-time horizon. It is to be noted that during the period of the data used, the world witnessed one of the greatest financial crises of all time which quite likely might have changed the dynamics of the markets. Another reason for divergence between the out-of-sample performances may be due to the nature of the model fitting. That is, a model that is back tested to perfection and has a good in-sample performance can become sluggish and unresponsive to changes in the volatility and sudden shocks while a model which performs poorly in the in-sample fit might be more flexible and hence be able to accommodate changes in volatility dynamics and shocks. There might also be a trade-off between fitting the model to the in-sample data and the model alertness to new inputs. An important finding is not only limited to the different ranking of the models when using different loss functions, but also how dramatically it can differ. It is quite a contrast that one loss function suggests that a particular model is the worst and another loss function ranks that same model to be the best. All through this research and taking note of the Akaike Information Criterion (AIC) for the GARCH, EGARCH, FIGARCH and TGARCH models, it was concluded that the FIGARCH modeling technique is more preferred for evaluating daily return series for Dangote cement Plc., Flourmill Plc., Nestle Plc. and Unilever Plc. while the TGARCH modeling technique is more preferred for daily return series for Guinness Plc. during the period of this study, (see table 3.6). The evidence of long memory in volatility across the indices suggests that FIGARCH model adequately describes the persistence than the conventional GARCH models. Therefore, in the backdrop of the present study, long memory models such as FIGARCH are recommended for volatility forecasting. The use of high frequency data and individual stocks composing different indices for further analysis would clarify the dynamics of market and explain interaction between volatility persistence and market microstructure variables.

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Authors' Conflicts of interest. None

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JOSEPHINE NNEAMAKA ONYEKA-UBAKA*

DEPARTMENT OF STATISTICS, UNIVERSITY OF LAGOS, AKOKA, LAGOS STATE, NIGERIA.

Email: jonyeka-ubaka@unilag.edu.ng

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KINGSLEY CHIMEZIE OHANMO

DEPARTMENT OF STATISTICS, UNIVERSITY OF LAGOS, AKOKA, LAGOS STATE, NIGERIA.

Email: kohanmo@aislagos.org