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IMPULSIVE SALE-PROFIT MODEL

BENJAMIN OYEDIRAN OYELAMI* AND SHEILA AMINA BISHOP

Abstract. This paper considers optimization problems that arise from mathematical economics involving the impulsive sale-profit model of a company producing some items in a mechanically independent non-competitive market situation. The sale and profit profiles of the company are analyzed using a variable allocation of funds for sales promotions, minimizing the cost functional subject to impulsive control and Lyapunov Functional. We formulate the sales projection of the company using the 'parasitic control measure' and the impulsive-averaging technique. The result shows that the sale is bounded in a redundant market situation if some sale parameters are finite. The approximate solution and the error of approximation of the model exist.

1. Introduction

Impulsive differential equations (IDEs) are equations whose states of evolution display instantaneous changes in the form of jumps or shocks and occur as small perturbations (impulses) at periods (fixed or non-fixed) during the process of evolution [[2],[3],[7],[13],[16]]. The solutions of IDEs exhibit discontinuities, the impulsive moments describing them depend on some impulse sets and the dynamics of the evolutionary process characterizing them [[1],[3],[6],[12],[13],[17]]. These and many more make the study of IDEs more problematic when compared with classical differential equations. Nevertheless, the IDEs offer suitable apparatus for investigating the behaviours of many real-life processes [[6],[17]].

1.1. **Preliminaries.** We state some definitions and notations which will lay the background of the methodology.
The set $C([-h,0],\Re)$ is the space of continuous variables, where $\Re=(-\infty,\infty)$ and $\Re^+=[0,\infty)$.

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* Correspondence.

Let $K = \{a(r) : a \in C([-h, 0], \Re)$ be monotonically increasing in r and $\lim_{r\to\infty} a(r) = \infty \}$.
The interval $D_{-h} = [-h, 0] \in \Re, h > 0$ is arbitrary.
$PC(D_{-h})$ denotes the function space $x : D_{-h} \to \Re^p$ such that,
$x(t)$ is continuous $\forall\, t \in D_{-h}, t \neq t_k,\ \max_{t\in D_{-h}} |x(t)| < \infty$ and

$$\lim_{t\to t_k+0} x(t) = x(t_k + 0) \text{ if } t_k \in (-h, 0],$$
$$\lim_{t\to t_k-0} x(t) = x(t_k - 0) \text{ if } t_k \in (-h, 0] \text{ exist, and}$$
$$x(t_k - 0) = x(t_k)$$

The sets D_1 and Ω are defined as follows:

$$D_1 = \left\{ z_0(z_k) \in \Re^p : |z_0(z_k(s))| \leq \alpha_0(s)e^{-\alpha s}, \alpha = \text{const} > 0 \right\} \textit{ and}$$

$$\Omega = \left\{ W(s,t) \in PC(\Re^+, \Re^+) : \left|W(t,s)\right| \leq l_0(s)l_1^{-1}(t)e^{-\alpha(t-s)} \text{ for } l_0(.), l_1(.) \in K \text{ and } \alpha = \text{const} > 0 \right\}$$

Next, we let $\eta = \sup_{x\in D} d(x, \text{diam } \Omega)$, diam $\Omega < \infty$, and $d(.,.)$ be Hausdorff's metric.

Definition 1.1. The sale vector $S_k(t)$ is mechanically dependent on time $t \in [-h, T], h > 0$ with respect to q if for some q, $S_k(t) = F(S_q(t))$ for some function $F(.)$ if $k \neq q, k = 1, 2, .., q$. That is, $S_k(t)$ cannot be expressed in terms of $S_q(t)$ for $k \neq q$, otherwise it is mechanically independent.

We consider the Bainov -Zabraiko-Kostadinov (BZK) function (See [[9],[11],[14]]) which is defined as

$$W(t,s) = \begin{array}{c} u(t,s), t_{n-1} < s \leq t \leq t_{n-1} \\ u(t,t_n)Q_n u(t,s_n), t_n < s < t < t_{n+1} \\ u(t,t_n)\prod_n Q_j u(t_j, t_{j-1})Q_k u(t_k, s), t_{n-1} < s < t_k < t_{n+1} \leq t_{n+1}, \end{array}$$

where $u(t)$ is the solution of

$$\begin{array}{c} \dot{u}(t) = A(t)u(t), t \neq t_k \\ u(t_k + 0) = Q_k u(t_k - 0) \\ 0 < t_0 < t_1 < ... < t_k, \lim_{k\to\infty} = \infty, \end{array}$$

$u(t,s) = u(t)u^{-1}(s)$ for $t, s \in \Re^+$.

1.2. **Literature Review.** It is important to note that the prices of fossil oil and other produce in the international market are impulsive because of the swift rise and fall in their prices within short periods. In recent times, the outbreak of covid-19 is making prices of goods and services to be impulsive. Also, the profits made on investments fluctuate in a yo-yo way [[11],[12],[14]]. Thus, impulse models are more appropriate for studying the prices of produced commodities from an industrial set-up. The model considered here is a deterministic model with potential applications in the study of investments in some fields of human endeavour that can be studied using IDEs. There are other types of impulsive and classical stochastic models for financial investment (see[[4],[11],[16]]).
The motivation for this study is to initiate an impulsive sale-profit model that can be applied in mathematical economics. The model assumes that the company producing some items operates in a mechanically independent non-competitive

market. The goals of the company are to optimize the sales of its products and minimize overhead production costs.
For the company to realize some of these goals, the following factors are considered: the government's policies on taxation, e.g. value-added tax (VAT), importation and exportation tariffs and other levies. Also, the sale promotion cost, bonus to customers and advertisement cost affect the company's profit. These factors are not overall exhaustive; this is to avoid building bulky models and the complexities associated with such models.
The model studied will consider the effect of taxation and advertisements on the sales of the products. Moreover, we assume that the taxation function is impulsive because of its swift upward or downward adjustment, which depends on the government tax policy.
The model studied will consider the effect of taxation and advertisements on the sales of the products. Moreover, we assume that the taxation function is impulsive because of its swift upward or downward adjustment, which depends on the government tax policy. Furthermore, the prices of products behave impulsively in most developing economies due to control by the market forces of demand and supply and sudden price adjustments by the company's dealers or middle marketers. The model will utilise the impulsive price function formulated via the impulsive neutral differential equations. The 'neutral nature' is justified since the past performance of the company is needed for making future projections on the sale of products.
This model is a typical optimization problem arising from the impulsive control system containing 'maximum', which is a relatively new area of research. Therefore, the problem will be analyzed using quantitative results in [[2],[5],[6],[7],[8],[14]] and other proposed results.

2. Materials and Methods

2.1. **The Sales profit model.** Considering the model equations

$$\frac{dS_k(t)}{dt} = \left(\frac{a_k}{m_k}A_k\left(t\right)-1\right)S_k\left(t\right)+b_kA_k(t)-c_kV_k-\frac{d_k}{P_k(t)}, t \neq t_k \tag{2.1}$$

$$S_k(t) = X_k(t), S_k(t) = X_k(t), t \in [-h, 0], h > 0 \tag{2.2}$$

$$S_k(t) = (1+{}_k)S_k(t), k = 0, 1, ... \tag{2.3}$$

$$f_kS_k(0)+g_kS_k(T) = h_k \tag{2.4}$$

$$M_k = \max_t S_k(t), t \in [-h, 0], h > 0 \tag{2.5}$$

Choose $P_k(t_k+0)$ so that

$$P_k(t_k+0) = \left(1 + \sum_{t_0<t_k<t} d_k^* V_k(t) P_k(t_k)\right) \tag{2.6}$$

for times $\{t_k\}, k = 1, 2, 3, ...$ such that
$0 < t_0 < t_1 < t_2 < \cdots < t_k < t_p < T$,
where $V_k, P_k, S_k \in PC([-h, T]), T \in [0, +\infty)$.
$S_k(t)$ denotes the sale of the k item at the time t ;
$A_k(t)$ is the amount spent on sale promotions of the product k at t, V_k is the tax or levy the company is required to pay for the procurement of raw materials.
$P_k(t)$ is the price of the product at time t.
a_k is the projected sale promotion rate.
b_k is the effective sale promotion rate constant.
c_k is the taxes coefficient.
d_k is the price index coefficient.
f_k, g_k and h_k are some relevant commercial constants for $k = 1, 2, ..p$.
T is the period for the sale of the items.
Note: if $A_k\,(t) > \frac{b_k}{a_k}, k = 0, 1, 2, 3, \quad . \ . \ . \ .$, the company will blossom.

3. Result

Assume

$$\widetilde{A}(t) = a_k A_k(t) - I; e_k U_k = e - C_k V_k - \frac{d_k}{P_k(t)} \tag{3.1}$$

$k = 0, 1, 2, \cdots$ in (2.1)
Then, the model equation becomes an impulsive control system whereby U_k represents all the control prices that affect the sales of the products.
We note that if VAT increases and $P_k(t)$ decreases, then V_k decreases as time progresses.
In Practice, if the square matrix $\widetilde{A}(t)$ is autonomous, then, there is always disbursement of funds for sale promotions at some periods. Hence, the solution to the model can be derived by slightly modifying the results in [[1],[7],[12]].

$$S_k(t) = B_k^{-1}\left(\exp \widetilde{A}(T) \prod_{t_k\in\delta_k} (I+\varphi_k) g_k - f_k \int_0^T \prod_{t_k\in\delta_k} (I+\varphi_k) \exp \widetilde{A}(T-s) \cdot H_k(s) ds\right) + \int_0^t \prod_{t_k\in\delta_k} (I+\varphi \tag{3.2}$$

where

$$B_k = d_k + f_k \exp \widetilde{A}(t) \cdot \prod_{t_k\in\delta} (I+\varphi_k)$$

$$H_k(t) = d_k A_k(t) + e_k u_k(t)$$

$$\delta_k = \{t_k \in (0, +\infty) : t_0 < t_k < t, k = 1, 2, \cdots\}$$

3.1. **Variable Allocation of Funds for Sale Promotions.** If $\widetilde{A}(t)$ in (4.1) is non-autonomous (steady increment or decline in fund allocation) for sale promotion purposes, then the solution of (3.1-3.4) becomes

$$S_k(t) = W(t,0)\left[g_k - d_k + \int_0^T W(s,t)H_k(s)ds\right] + \sum_{0<s_k<t_k\in_k} W(s_k,t_k)(1+_k)S_k(t). \tag{3.3}$$

Here, $W(s,t)$ is the BZK.

3.2. **Optimal Criteria.** Let the cost functional (performance index) for the production of the items be

$$J_c = \frac{1}{2}S_k^*(t)FS_k(t) + \frac{1}{2}\left[\int_0^\infty S_k^*(s)FS_k(s) + u_k^*(s)Ru_k(s)\right]ds \tag{3.4}$$

where F and R are $p\times p$ positive definite and $p\times p$ semi positive definite matrices respectively, and $S_k^*(t) = S_k^T(t)$.
To minimize the cost of production, minimize J_c subject to the problem in (2.2)-(2.6). To achieve this, we consider two cases using some kinds of Lyapunov functions:
Case 3.1. (Autonomous):
Let $U_k = -FS_k(t)$ and define

$$V(S_k(t)) = S_k^*(t)G_kS_k(t), \tag{3.5}$$

where

$$G_k = \begin{cases} JG^kJ^{-1} \text{ if } t\in\Re^+ \\ Z^*G^kZ^{-1} \text{ if } t\in[-h,0] \end{cases}$$

J is a $p\times k$ Jordan matrix, Z is a $p\times k$ simpletic group and G is a positive definite matrix. Then

$$V = S_k^*(t)[(\widetilde{A} - d_kF)^*G + G(\widetilde{A} - d_kF) + \alpha_k(A_k^*G + GA_k^*)]S_k(t) \tag{3.6}$$

$$\begin{aligned} V(S_k(t_k^+)) &= S_k^+(t)\left[G + \varphi_k^+P + P\varphi_k + \varphi_k^+P\varphi_k\right]S_k(t) \\ &= S_k^+(t)GS_k(t) - S_k^+(t)\varphi_kS_k(t) \end{aligned} \tag{3.7}$$

Then, the following matrix equations are obtained from equations (3.6) and (3.7):

$$(\widetilde{A} - d_kF)^* + (G\widetilde{A} - d_kF) + \frac{1}{2}(F - F^*RF) = 0 \tag{3.8}$$

$$\varphi_k^*G + G\varphi_k + \varphi_k^*G\varphi_k - \varphi_kG = 0 \tag{3.9}$$

It can be established from Lemma 1 in [[7]] that

$\dot{V} \geq \frac{1}{2}S_k^*(t)(F - \dot{F}^*RF)S_k(t) \geq -\lambda S_k^*(t)GS_k(t)$ and

$$V(S_k(t_k^+)) \geq (1+\eta_k)V(S_k(t)),$$

where the constants η_k and λ are positive. Therefore, (2.1) is δ-controllable [[9]] and its equilibrium point is asymptotically stable [[6],[17]].

Thus, if η_k and λ are chosen in such way that

$$(1+\eta_k)\exp(-\lambda(t_{k+1}-t_k)) \geq 0,$$

then

$$\lim_{t\to\infty} V(S_k(t)) = 0$$

and

$$\begin{aligned} J_c &= S_0^*(t)P_0S_0^*(t) + \tfrac{1}{2}S_k^*(t)FS_k(t) + \textstyle\sum_{k=0}^{\infty} M_kS_k(t) \\ &\leq V(S_0(t)) = S_0^*(t)GS_0^*(t) \end{aligned}$$

Therefore J_c is minimal if

$$J_c = S_0^*(t)P_0S_0^*(t) + \frac{1}{2}S_k^*(t)FS_k(t) + \sum{}_{0k}GS_k\,(t) = S_0^*(t)GS_0^*(t) \tag{3.10}$$

Moreover, G are the matrix solutions of (3.9) and (3.10) respectively.

Case 3.2. (Non-Autonomous):

$$Let V(S_k(t)) = S_k^*(t)\cdot\exp G(t)\cdot S_k(t). \tag{3.11}$$

and $U_k = -F(t)S_k(t)$. Then the optimal control policies for the sale-profit equation can be obtained by solving the following:

$$A^*\cdot\exp G + G\cdot\exp G + \exp G\cdot A = -I \tag{3.12}$$

$$\exp G\cdot(b_kA_k + e_kU_k) = 0 \tag{3.13}$$

I is the Identity matrix.

Since analytic solutions to (3.12) and (3.13) are almost impracticable, numerical methods like Newton-Krantowski can be employed to find the solutions.

Since $\exp G$ can be approximated by the matrix $I+G$ with $||G|| \leq 1$,
the iterative matrix equation for (3.12) becomes

$$G_{k+1} = G_k + (I+G_k)^{-1} - A - (I+G_k)\widetilde{A}(I+G_k)^{-1}$$

Subject to $(I+G_k)(b_kA_k+e_kU_k) = 0$ and it is easy to show that the control is $u_k = \frac{-b_k}{e_k}A_k(t)$.

For example take $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, and $G_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Then we find that $G_1 = \begin{pmatrix} 3/2 & 0 \\ 0 & 1/2 \end{pmatrix}$.

3.3. **Projection Using Parasitic Measures.** In real-life situations, the possibility of a company making projections by defaulting on the error in taxation or tax relief can happen by introducing "parasitic" control measures to reduce taxation effect on the company or underpayment of taxation. In this situation, it is best to analyze the problem using the impulsive analogue of averaging technique (See [[2], [5]- [8]] and the references therein).

Subsequently, we will apply the averaging methods to the Sale-Profit model. We note that the system (3.1)-(3.5) is equivalent to the following average equation

$$\overset{\bullet}{S}_k(t) = Z_0(S_k) + \in_k U_{0k}, t \neq t_k \tag{3.14}$$

$$S_k(t) = \varphi_k(t), if\ t \in [-h, 0], h > 0 \tag{3.15}$$

$$S_k(t_k^+) = I_0(S_k) \tag{3.16}$$

$$d_k Z_{00} + f_k\, z_0^T = g_{ok} \tag{3.17}$$

where

$$z_0(S_k) := lim_T \frac{1}{T} \int_0^T \left[A(s)S_k(s)\tilde{} - d_k A_k(s) \right] ds \tag{3.18}$$

$$T_0(S_k) := \lim_{T\to\infty} \frac{1}{T} \sum_{t_k \in \delta_k} \varphi_k(S_k(t_k)) \tag{3.19}$$

$$z_{00} := z_0(0), z_0^T := z_0(T)$$

Here $\in_k, k = 1, 2, 3...$ are the parasitic terms or taxation errors which account for default taxation or error due to under payment of tax.
And the solution is given by

$$z_k(t) = z_k(0) + \int_0^t z_0(z_k(s))ds + \in_k \int_0^T u_{0k}(z_k(s))ds + \int_0^T I_0(S_k(s))ds \tag{3.20}$$

where

$$z_k(t) = \varphi_k(t), \text{ if } t \in [-h, 0], h > 0.$$

3.4. **Projection under a Redundant Market Situation.** In a redundant market situation, with no advertisement policy and the economy is free, we have $C_k = 0$, $\alpha_k = b_k m_k$. Then the equation (3.1) reduces to

$$\dot{S}(t) = -S_k(t) - \frac{d_k}{P_k(t)}, t \neq t_k, 0 < t < T \tag{3.21}$$

and we obtain the following solution.

$$\begin{aligned} S_k(t) = V(t)S_0 \prod_{l=1}^{k+1} (I + \varphi_k) + \textstyle\int_{-h}^0 V(t-\tau-s)S_t(s)ds \\ - d_k \textstyle\int_{t_0}^t z_k(\tau) \frac{d\tau}{P_k(\tau)}, \end{aligned}$$

where

$$z_k(\tau) = V(\tau) \prod_{k=1}^{k+1} (1 + \varphi_k) - \int_{-h}^0 V(\tau - s)S_k(s)ds. \tag{3.22}$$

Where

$$\left. \begin{array}{c} \frac{dV(t)}{dt} = V(t-\tau), t > 0 \\ V(t) = 0, t \in [-h, 0], V(0^+) = 1 \end{array} \right\} \tag{3.23}$$

If $V(t) = e^{-\frac{t}{\tau}} u(t)$, then $u(t) = 1 + e[1 - e^{\overline{-t}}]$, see [[6]].
If $S_{\max}(t) > S(t) > \frac{d_k}{P_k(t)}$, then the company will blossom with $S_{\max}(t)$. This can be obtained by estimating the upper bound for $S_k(t)$ in equations (3.22) and (3.23).

We can also show by iteration that

$$\begin{aligned} S_{k+1}(t) = S_0 &\left(1+\tau e[1-e^{-\frac{t}{T}}]\right) e^{-\frac{t}{T}}(1+\varphi_0)^n \\ &\times \left[\int_{-h}^{0}(1+\tau e\left(1-e^{-\frac{t-\tau-s}{\tau}}\right)\right] e^{-\frac{t-\tau-s}{\tau}} S_k(s)ds \\ &- d_k \int_{-h}^{0} \frac{z_k(\tau)}{P_k(\tau)} d\tau \end{aligned} \tag{3.24}$$

Suppose that $S_k\left(t\right)S\left(t\right)$, $z_k\left(t\right)z\left(t\right)$ and $P_k(t) \to P(t)$ as $k \to \infty$ then $S(t) = \lim_{k\to\infty} |S_k(t)| = \max_k |d_k| \int_{-h}^{0} \frac{|z(\tau)|}{|P(\tau)|} d\tau, \tau \leq t.$
Showing that under a redundant market, the sale of the products is bounded, and this can be established by estimation of the upper bound for (3.18) as follows:
Let $E = |S_0|\left(1+\tau e\right)^2\left(1+|\varphi|_0^n\right), R = \max_k |d_k| \int_{-h}^{0} \frac{|z(\tau)|}{|P(\tau)|} d\tau$
and $F = \frac{E}{|S_0|}$.
We can show that

$$|S_1(t)| \leq E+R, |S_2(t)| \leq F+hE+h(E+R)R$$

$$|S_3(t)| \leq (1+h)F+h^2E+R(1+h(1+h))$$

$$\vdots\, |S_4(t)| \leq (1+h)E^2+h^2EF+RF(1+h(1+h))+R$$

$|S_n(t)| \leq M, M$ is a constant.
Therefore the sale is bounded above for all k and for finite values of $h, \max_k |d_k|, |\tau|, |\varphi_0|$ and for $\int_{-h}^{0} \frac{|z(\tau)|}{|P(\tau)|} d\tau < \infty$

Proposition 3.1.
Assume that there exist $l_i(t) \in K, i = 2,3,4$ with m, η and T as constants so that

(1) $|W(t,0)g_k - Z_k(0)| \leq l_2(t)$
(2) $|H_k(t)| \leq \frac{l_3(t)}{\eta \max[1,kT,|d_k|]}$
(3) $|S_k(t)| \leq \frac{l_4(t)}{\max[1,\eta m T,|d_k|]}$

$$|U_{0k}(Z_k(S_k)| := O(m)$$

(4) $q_k := l_3(t)+l_4(t)$,$l_2(t)+q_k := r_4(t)$ $r_4(t)+Z(0) = \delta_{0k}(t)$ $|W(t,0)| + |\delta_{0k}(t) < \tau|$.

Then, the solution of the problem (2.1)-(2.5) can be approximated by (3.14)-(3.20) with

$$|S_k(t) - z_k(t)| = O(\tau^{\vartheta})$$

as the error of approximation and τ as the error of computation of order ν .
Proof.
The proof is an extension of the method employed in Theorem 1 of the reference [[2]].

4. Conclusion

We study a sale-profit model with a variable allocation of funds for sale promotions. We were able to show that the cost of production can be minimized using the impulsive control Lyapunov function. The solution to the model is bounded whenever the sales parameters are finite. This result is in affirmation with the results obtained in [[11]]: that the return on investment is always bounded in the long run no matter the business strategies used. Lastly, for further study, we recommend that the sale-profit model should be extended to the case where variable time is used instead of simulation time.

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Dedication: To the loving memory of late Prof. Anthony U.Afuwape, a mathematical giant whose support and encouragement in the Nigeria Mathematical Society spurred us on before his demise. His sense of humor and love for the young mathematicians will forever remain in our minds.

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Benjamin Oyediran Oyelami*
Department of Mathematics, National Mathematical Centre, Abuja, Nigeria
E-mail address: boyelami2000@yahoo.com

Sheila Amina Bishop
Department of Mathematics, University of Lagos, Nigeria
E-mail address: asbishop@unilag.edu.ng