Unilag Journal of Mathematics and Applications,
Volume 3, (2023), Pages 35--52.
ISSN: 2805 3966. URL: http://lagjma.edu.ng

MARKOV SWITCHING DYNAMIC REGRESSION MODEL: APPLICATION TO FINANCIAL DATA

ROSEMARY UKAMAKA OKAFOR* AND JOSEPHINE NNEAMAKA ONYEKA-UBAKA

ABSTRACT. Hidden Markov Models are used to study occurrences where a portion of the phenomenon is observable while the rest is unobservable. The model consists of two sets of random variables, namely, unobservable and observable random variables. Markov models are usually used to model the unobservable variable part of the hidden Markov model; a time-series regression model is used to model the observable part of the model. Then, the two random variables produce a regime-switching model known as the Markov-switching dynamic regression model with a distinct probability distribution. The parameters of this resulting model can be estimated using the maximum likelihood estimation method, and the characteristics of the regression model vary with the presiding regime of the model. The regression objective of this work is to explain the variability in the Nigerian monthly inflation rates using the variability in the average monthly United States dollar exchange rates. The correlation between the two datasets is determined using the least squares regression model and the Markov switching dynamic regression model. The Markov switching regression (Regime 2) model represents the high-variance regime. It explains the regression objectives more than the OLS regression model, as shown in the results. Consequently, it shows that when the Markov state model is in the high variance regime, Nigeria's economy shows a recession. The results depict that the regime-switching model outperformed the single regime model in capturing the properties of the data from the comparative analysis.

1. INTRODUCTION

A hidden Markov model (HMM) is a stochastic process with two random variables: the unobservable state sequence and the observable sequence of independent random variables. HMMs are used to study occurrences where a portion of the phenomenon is visible while the rest is invisible. The observed part is to be used to estimate the effects of the unobserved part. Markov-switching models are excellent for series that transition over a finite set of unobserved states, which allows the process to evolve differently in each state. This transition is assumed to follow a random

---

2010 *Mathematics Subject Classification*. Primary: 22E30. Secondary: 58J05.

Keywords and phrases. Hidden Markov, Observable variable, Unobservable variable, Regime-switching, Distinct probability, Regression model

Submitted: July 25, 2023. Revised: December 13, 2023; February 5, 2024. Accepted: March 23, 2024.

*Correspondence

Markov process, where both the timing of the transition from one state to another and the intervals between the changes are also random processes.

Hidden Markov Models, also known as Markov Regime Switching Models, are widely used tools for modeling sequences of dependent variables. For more details on Markov switching models, see Chung-Ming [1], Jurafsky and Martin [2]. Researchers in different disciplines have utilized Markov regime-switching models in their works: Mor et al. [3] provide a comprehensive description of research on hidden Markov models and their applications. Pomorski and Gorse [4] proposed an approach that combines the two-state Markov switching regression with an adaptive moving average. Kartal et al. [5] explore the regime-switching effect of the pandemic on Turkey's stock market index. Zivkov et al. [6] examine the impact of oil price changes on inflation in eleven central and eastern European countries using a wavelet-based Markov switching approach. Tesfamichael and Shiferaw [7] use the Markov-switching (MS) approach to predict the Normalized Difference Vegetation Index (NDVI) at a biome spatial scale. Costa and Kwon [8] introduce a Markov regime-switching framework for risk parity optimization. Alvarez et al. [9] present a statistical theory that offers asymptotic approximations for single inferences of filtered and smoothed probabilities from time series characterized by Markov-switching dynamics. Carstensen et al. [10] utilize a Markov-switching dynamic factor model based on six leading business cycle indicators specific to Germany. Moutinho et al. [11] examine the relationship between energy commodity prices and wholesale electricity prices in the Iberian electricity market.

Kim et al. [12] delve into the mixture of quantile and Markov-switching regression, with the resultant model offering flexibility in handling structural or time-varying parameter models. Stillwagon and Sullivan [13] study Markov switching models of exchange rates, taking a data-driven approach to identify the number of regimes rather than assuming a fixed number. Hwu et al. [14] study an N-regime Markov-switching model that incorporates an endogenously determined latent state variable influenced by the disturbance term within the model itself. Adam et al. [15] introduce a new class of adaptable latent-state time series regression models that offer greater flexibility than traditional Markov-switching regression models. Rahman et al. [16] show the relationship between financial development and economic growth in Pakistan using the Markov Switching Model. Anser et al. [17] investigate the relationship between air pollutants and COVID-19 cases in 39 highly impacted countries using a Markov switching regime regression model and an innovation accounting matrix. Shah et al. [18] employ advanced econometric methods and Markov switching regression to examine the effect of COVID-19 on renewable electricity generation in Denmark. Degras et al. [19] propose new statistical techniques for computational challenges in Markov switching models.

This paper describes an approach with a hidden Markov model applied to occurrences where a portion of the process is observable or visible while the rest is unobservable or hidden. This approach assumes that the dependence between the observed and unobservable variables corresponds to a linear model. A Markov-switching dynamic regression (MSDR) model is the resultant model when a regression model is used to simulate the observable variable and a regime-switching Markov model to simulate the unobservable variable. The switch in the regimes is a non-observable latent variable, and the visible variable is conditioned on it, which results in different sets of parameters for each state. A case where the observations are assumed to be finite discrete variables and switching of regimes or states is allowed, which gives a discrete probability density within each state, is adopted. The parameters of the MSDR model were estimated using the maximum likelihood estimation (MLE) method, and the dissection of the maximum likelihood estimation method for the Normal probability distribution density was analyzed. In a statistical model, parameter estimation involves determining the optimal values of the parameters of the population distribution under the assumption that the model is a descriptive mechanism that creates data with unknown parameters. The MLE approach described the optimal values of the transition probabilities and the regression coefficients that would maximize the joint probability density of modeling the data set. The Nigerian monthly inflation rates and the average monthly USD exchange rates represent the data set. The regression goal is to explain the variance in inflation rates using the variability in USD exchange rates. The correlation between the two data sets was determined using the least squares regression model and the Markov switching dynamic regression model. The parameters of the Markov switching regression model were estimated using the maximum likelihood estimation method, and comparisons were made between the models using best-fit statistics.

2. MATERIALS AND METHODS

A hidden Markov model is a stochastic non-linear time series model in which the system assumes a Markov process with unobserved states. This model involves different equations that characterize the time series dynamics in different regimes. The model can capture more complex dynamic patterns by enabling transitions between states. The dominant characteristic of the Markov switching model is that the switching mechanism is to be controlled by an unobservable state variable that follows a first-order Markov chain.

The effect of the unobserved part of the hidden Markov model is estimated using the observed portion of the process. Consequently, it represents a mixture of two random processes: the observable random variable for the visible part and the hidden random variable for the unobservable part. A time series regression model is used to model the unobserved process, while a Markov model is used to model the unobserved portion of the process.

2.1 Hidden Random Variable

Using a regression model that is a mixture of two random variables: the observable random variable y_t, used to represent the observable pattern at each time step t and a hidden random variable S_t which is assumed to change its state or regime. Since the observed pattern is affected each time the regime changes, therefore, a change in the value of S_t affects the mean and the variance of y_t.

Let's assume a two-regime Markov process characterized by the hidden random variable S_t in this work. S_t is a hidden random variable because the point it changes its regime is unobservable and influences the observed trend which is characterized by the random variable y_t.

The study explains the variability in the monthly inflation rate in Nigeria using the average monthly United States dollar exchange rate. The analysis utilized the least-squares regression model and the Markov-switching dynamic regression model. The latter, particularly in Regime 2, explains the regression objectives better than the ordinary least squares regression model. The results show a recession in the economy when the Markov state model is in the high-variance regime. The conclusion drawn from the results indicates that the regime-switching models, especially in capturing the properties of the data, mostly outperform the single-regime models, as was proven by many authors in the literature. The work suggests incorporating regime-switching dynamics in explaining the relationship between inflation and exchange rates.

2.1.1 The State transition probability

The state transition probability P, is the probabilities of transition to the next state which are conditional upon the current state.

The state transition probability of a 2-state Markov process in matrix form can be expressed thus:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{22} & P_{22} \end{bmatrix}$$

2.1.2 The state probability vector

The state probability vector $\{\pi_t\}$ is the unconditional probability of being in a certain state at time *t*. For the 2-regime Markov random variable, the state probability distribution π_t is given by the following 2-element vector:

π_t = P(S_t=1), P($S_t = 2$).

If the initial probability distribution for S_t is taken as π_0, then π_t is

$$\pi_t = \pi_0 P^t \qquad (2.1)$$

2.2 Observable Random Variable

Let the regression model for the observed variable y_t be:

$y_t = \hat{\mu}_t + \varepsilon_t \quad , \quad \varepsilon_t \sim N(0, \sigma^2)$ (2.2)

where $\hat{\mu}$ is the regression model's prediction of the observed at time *t* and ε_t is the random error.

The equation above means that the observed y_t at any time *t*, is the sum of the modeled mean $\hat{\mu}_t$ or predicted mean and the residual error ε_t provided the Least squares regression assumptions hold, which is that the variables have an additive relationship with each other, the residual error ε_t is normally distributed with zero mean and the data have constant variance σ^2 (homoskedastic).

Since $\hat{\mu}_t$ is the predicted value of the regression model, $\hat{\mu}_t$ is the link fuction denoted as $\eta(.)$ which specifies the link between the random and the systematic components. From the Generalized Linear Model, It indicates how the expected value of the response, $E(y_i) = \mu_i$ relates to the linear combination of explanatory variables $\mathbf{X}'_i\boldsymbol{\beta}$. See Sachin [20].

The link function $\eta(X_t, \hat{\beta})$ is the identity link function $\eta(.) = \hat{\mu}_t$, for classical linear regression models of continuous outcomes with normal distribution, where x_t is the regression variables and the vector, $\hat{\beta}$ is the fitted coefficients, thus:

$\hat{\mu}_t = x_t \,.\, \hat{\beta}$ (2.3)

where $\hat{\mu}_t = \hat{\beta}_0 + \hat{\beta}_1 x_{1t} + \hat{\beta}_2 x_{2t} + \cdots + \hat{\beta}_m x_{mt}$

Then, $y_t = x_t \,.\, \hat{\beta} + \varepsilon_t$ (2.4)

If y_t is assumed to be normally distributed with mean $\hat{\mu}_t$ and constant variance σ^2. Then, the probability density function of the observed, y_t is thus:

$f\,(\mathbf{Y} = y_t |\, \mathbf{X} = \boldsymbol{x_t} \,;\, \hat{\boldsymbol{\beta}}\,) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - \hat{\mu}_t}{\sigma}\right)^2}$ (2.5)

The above equation reads that the probability density of the observed y_t at time *t*, conditioned upon the regression variables vector $\boldsymbol{x_t}$ and the fitted coefficients vector $\hat{\boldsymbol{\beta}}$ is normally distributed with a constant variance σ^2 and a conditional mean $\hat{\mu}_t$. To show how the modeled mean $\hat{\mu}_t$ and variance σ^2 of the visible random variable y_t are influenced by the hidden Markov process S_t, the two random variables is combined to obtain a Markov switching dynamic regression model. See Sachin [21] for details.

2.3 Markov Switching Dynamic Regression (MSDR) model

A Markov switching model is constructed by combining two or more dynamic models using a Markovian switching mechanism.

Let's define the regression model as:

$$y_t = \hat{\mu}_{t S_t} + \varepsilon_t \qquad (2.6)$$

Where the predicted mean of the model, $\hat{\mu}_t$ changes depending on particular state the underlying Markov process variable S_t is resident, at time *t*. The predicted mean $\hat{\mu}_{t S_t}$ can also be expressed as the output of the identity link function *η(.)*:

$$\hat{\mu}_{t S_t} = \eta(.) \qquad (2.7)$$

For this model, since there is one regression variable, the link fuction η(.) is thus:

$$\hat{\mu}_{t S_t} = \eta(.) = \mathbf{1} \, . \, [\hat{\beta}_{0 S_t}] = \hat{\beta}_{0 S_t} \qquad (2.8)$$

where **1** is the identity matrix.

If it is assumed that the Markov process operates over the set of *k* states [1,2,3,…*j*,…,*k*], then, it is expressed as:

$$\hat{\mu}_{t\,j} = \eta(.) = \hat{\beta}_{0j} \text{ where } S_t = \text{j} \qquad (2.9)$$

The model's regression equation is therefore written as:

$$y_t = \hat{\mu}_{t\,j} + \varepsilon_t \qquad (2.10)$$

where y_t is the observed value, $\hat{\mu}_{t\,j}$ is the predicted mean when the Markov process is in state *j* and ε_t is the residual error of regression.

For the data set, it is assumed that S_t switches between two regimes 1 and 2. Thus for $\hat{\mu}_{t\,j}$:

$$\hat{\mu}_{t\,j} = \eta(.) = \hat{\beta}_{01} \text{ when } S_t = 1$$

$$\hat{\beta}_{02} \text{ when } S_t = 2$$

From the regression's equation $y_t = \hat{\mu}_{t_{S_t}} + \varepsilon_t$, y_t switches between two means; μ_{S_1} and μ_{S_2} as follows:

$$y_t = \hat{\beta}_{0_1} + \varepsilon_t \text{ when } S_t = 1 \tag{2.11}$$

$$y_t = \hat{\beta}_{0_2} + \varepsilon_t \text{ when } S_t = 2 \tag{2.12}$$

Thus, the corresponding two conditional probability densities of y_t are as follows:

$$f(\mathbf{Y} = y_t | \boldsymbol{x_t} = \mathbf{1} \,;\, \hat{\boldsymbol{\beta}} = \hat{\beta}_{0_1} \,;\, S_t = 1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - \hat{\beta}_{0_1}}{\sigma}\right)^2} \tag{2.13}$$

$$f(\mathbf{Y} = y_t | \boldsymbol{x_t} = \mathbf{1} \,;\, \hat{\boldsymbol{\beta}} = \hat{\beta}_{0_2} \,;\, S_t = 2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - \hat{\beta}_{0_2}}{\sigma}\right)^2} \tag{2.14}$$

But in real application, each observed random variable y_t, is defined by only one probability density. Therefore we use the Law of Total Probability to calculate the probability density. The law states that if event A can take place pair-wise jointly with either event A1, or event A2, or event A3, and so on, then the unconditional probability of A can be expressed as:

$$P(A) = P(A|A_1)P(A_1) + P(A|A_2)\,P(A_2) + \ldots + P(A|A_n)\,P(A_n) \tag{2.15}$$

Therefore, to calculate the unconditional probability density of observing a specific observed state y_t at time t for the two states, we have:

$$f\,(\mathbf{Y} = y_t) = \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - \hat{\beta}_{0_1}}{\sigma}\right)^2} P(S_t = 1) + \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - \hat{\beta}_{0_2}}{\sigma}\right)^2} P(S_t = 2)\right) \tag{2.16}$$

Equation (2.16) in summation form is thus:

$$f\,(\mathbf{Y} = y_t) = \sum_{j=1}^{2}\left[\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_t - \hat{\beta}_{0_j}}{\sigma}\right)^2} P\,(S_t = j)\right)\right] \tag{2.17}$$

Equation (2.17) is the unconditional probability density of Y under the influence of a 2-state Markov model and the probabilities; $P(S_t = 1)$ and $P(S_t = 2)$ are the state probabilities vector π_{t1} and π_{t2} of the 2-state Markov process:

$$\pi_t = [\pi_{t1} = \mathrm{P}(S_t = 1), \pi_{t2} = \mathrm{P}(S_t = 2)] \qquad (2.18)$$

To calculate the state probabilities, assume some initial conditions and use the following equation:

$$\pi_t = \pi_0 P^t \qquad (2.19)$$

where π_0 is the assumed initial value at $t = 0$ and P is the state transition matrix of the 2-state Markov process.

2.4 Estimation of the Model Parameters

Because it is required to specify the distribution of the underlying population under review, parameter values are required. The Maximum Likelihood Estimation (MLE) method, was used for the parameter estimation.

The MLE method selects the parameter values that make the observed data most probable under the assumed statistical model. Hence, to determine the model's equation, we find the optimal values for the five parameters that maximize the joint probability density of observing the entire data set.

These variables are: the state transition matrix P which are the transition probabilities P_{11} and P_{22}, the state specific regression coefficients $\hat{\beta}_{0_1}$ and $\hat{\beta}_{0_2}$ and the constant variance σ^2.

Therefore, using the likelihood method to maximize the joint probability density of observing the data set. The likelihood function is:

$$L(\widehat{\boldsymbol{\beta}}\mathrm{s}; \sigma^2 ; \mathbf{P} \mid \mathbf{Y}) = \prod_{t=1}^{n} f(\mathbf{Y} = y_t) \qquad (2.20)$$

where $L(\widehat{\boldsymbol{\beta}}s; \sigma^2 ; \mathbf{P}| \mathbf{Y})$ is the likelihood of observing $\mathbf{Y}$, n is the size of data set and $f(\mathbf{Y} = y_t)$ is the the probability density of observing y_t given β, P.

But the likelihood function and log likelihood function have their maxima at the same value and it is easier to find the maximum of the logarithm of the likelihood. The log likelihood is thus:

$$\ell(\widehat{\boldsymbol{\beta}}\mathrm{s}; \sigma^2 ; \mathbf{P}| \mathbf{Y}) = \sum_{t=1}^{n} \ln(f(\mathbf{Y} = y_t)) \qquad (2.21)$$

Therefore, for the Maximization Log-Likelihood of the parameters, the partial derivatives of the log-likelihood w.r.t. each parameter P_{11}, P_{22}, $\hat{\beta}_{0_1}$, $\hat{\beta}_{0_2}$ and σ^2 are set to zero, and the resulting system of equations solved. If regularity conditions are satisfied, the point where the likelihood is at maximum is a solution of the equations.

3. RESULT

The data sets used for this work are the Nigerian monthly percentage inflation rates and the United States Dollar exchange rates from January 2004 to April 2021. The source is the Central Bank of Nigeria website, with each variable totaling 208 observations. Part of the data fell into the pandemic era, but the devastation was not significant on the Nigerian market, and the effects of COVID-19 on the economy were negligible. The objective is to check the relationship between the inflation rates and the USD exchange rates at the prescribed period, then analyze the variance in inflation using the variability in the USD exchange rates. The Markov Switching Model in R was used to model the variables. The hidden Markov part of the Markov switching model helped to explain the variabilities. See also Ibn [22] for details.

To model the data set, a regression model where y_t, the observable random variable, is the inflation rate and S_t, the hidden random variable, assumed to change its state or regime by switching between regimes 1 and 2 is adopted.

From the regression's equation $\boldsymbol{y_t} = \boldsymbol{x_t} . \widehat{\boldsymbol{\beta}} + \varepsilon_t$, the random variables for the regression model are:

Y = Dependent variable (returns of Inflation rates)

X = Regression variable (returns of USD exchange rates)

3.1 Exploratory Data Analysis

The descriptive statistics of the two variables and their log returns are in Table 3.1. The time plots of the inflation and exchange rates are shown in Figures 3.1 and 3.2, respectively.

TABLE 3.1: Descriptive statistics of the random variables

	% Inflation Rates	%Inflation Rates_returns	Exchange Rates(Naira)	Exchange Rates_returns
Observations	208	207	208	207
Minimum	3.0	-1.0	118.70	-0.14
Maximum	28.2	0.5	494.70	0.14
Mean	12.0	-0.0	227.47	0.00
Variance	16.7	0.0	13128.47	0.00
Stdev	4.1	0.1	114.58	0.03
Skewness	0.8	-1.9	0.93	0.55
Kurtosis	1.5	17.1	-0.70	4.60

The basic statistics of the time series data of inflation and exchange rates show a non-zero mean and a high variance, which indicate non-stationarity for both inflation and exchange rates.

The time plots of inflation and exchange rates in Figures 3.1 and 3.2 also show non-stationarity. Both charts have regions of high and low volatility. These are upward and downward trends, which indicate variations over time. The periods of low volatility are designated as Regime 1, while those of high volatility are Regime 2 in this work. The ACF plots of the inflation and the exchange rates in Figures 3.3 and 3.4 also show non-stationarity of the time series.

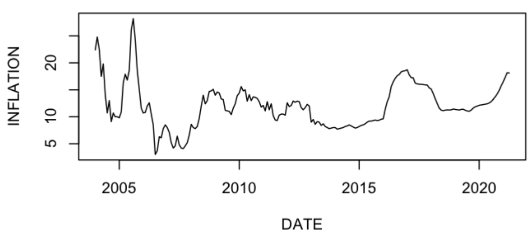

Figure3.1: Timeplot of Inflation Rates

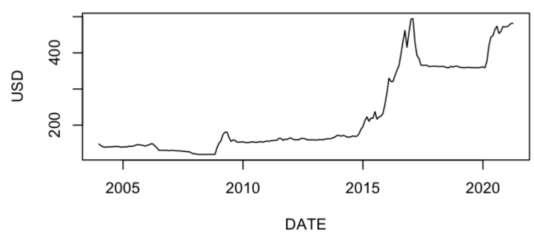

Figure 3.2: Timeplot of USD Exchange Rates

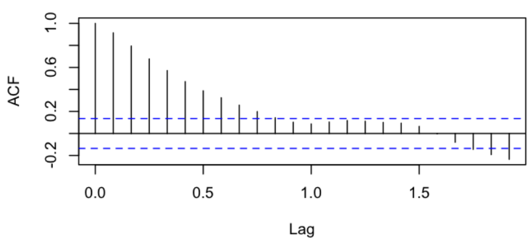

Figure 3.3: ACF plot of Inflation Rates

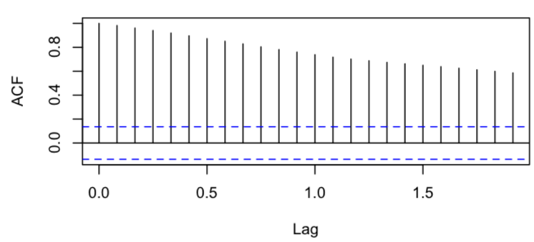

Figure 3.4: ACF plot of USD Exchange Rates

Thus, to make the process stationary, the time series is transformed to get the log returns of the two variables. The statistics of the log returns of the two variables shown in Table 3.1 indicates that the means are zero and, therefore, constant. The time-series plot of the log returns also confirms stationarity.

3.2 Regression Analysis

The regression equation is: $y_t = \beta_0 + \beta x + \varepsilon_t$

where y_t is the change in Inflation rate, β_0 is the intercept and βx is the coefficient of the change in USD exchange rate.

TABLE 3.2: Ordinary Least Square regression analysis results

β	Estimate	Std. Error	P-Value
β_0	-0.003995	0.009504	0.6747
βx	0.520168	0.269439	0.0549

Residual standard error: 0.1349 on 205 degrees of freedom

Multiple R-squared: 0.01786, Adjusted R-squared: 0.01307

F-statistic: 3.727 on 1 and 205 DF, p-value: 0.05492

From the Ordinary Least Square regression analysis results in Table 3.2, the model's residual standard error is 0.135. The regression equation is:

Change in Inflation = -0.003995 + 0.520168 × change in exchange rates + ε_t

It shows that when there is a 0.5 unit increase in value in the exchange rate, there is, on average, a difference of equal value in the inflation rate. The intercept and slope are the coefficients of the fitted model.

The regression diagnostics to check for the validity of the residual assumptions are in Figure 3.5.

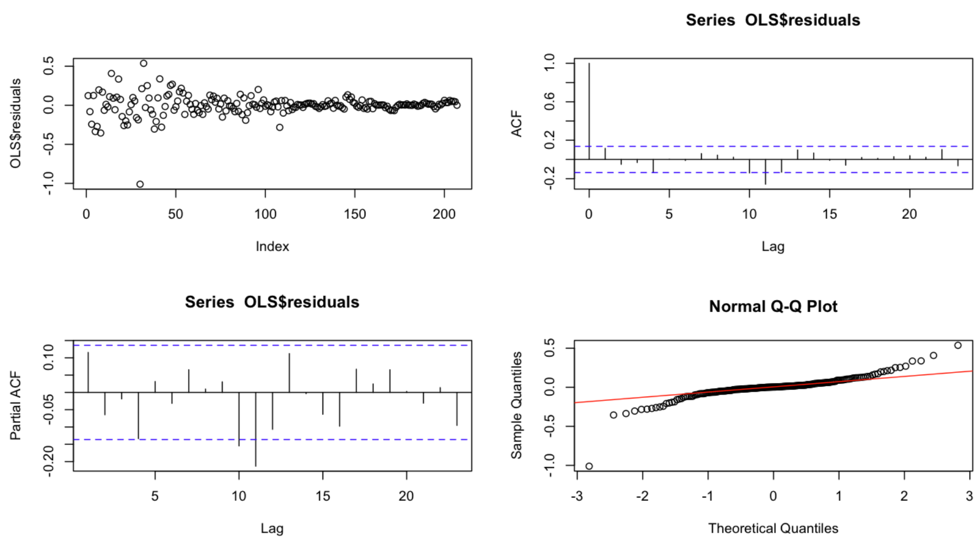

FIGURE 3.5: Plots of residuals regression diagnostics

3.3 Markov Switching Model

The time plots of the inflation and exchange rates show regions of low and high variance. To model the switches between high and low-variance states, we use a 2-state Markov switching dynamic regression model, where state 1 is the low-variance regime while state 2 is the high-variance regime. Since the Markov model operates over states [1, 2], the regression coefficients take the form of a [(m+1) X *k*] sized matrix of regression variables, where m is one and *k* is two in this case.

Depending on which Markov state or regime, the regression model coefficients will switch to the appropriate regime-specific vector $\hat{\beta}_j$ from $\widehat{\boldsymbol{\beta}}$s.

The MSDR model's equation is thus:

$y_t = \hat{\mu}_{t\,j} + \hat{\boldsymbol{\beta}}\text{s} + \varepsilon_{tj}$

where $\hat{\mu}_{t\,j} = \hat{\beta}_{0_j} + \hat{\beta}_{1_j} x_{1t}$

$$\hat{\boldsymbol{\beta}}\text{s} = \begin{bmatrix} \beta_{01} & \beta_{02} \\ \beta_{11} & \beta_{12} \end{bmatrix} = \begin{bmatrix} -0.0079 & 0.0007 \\ 1.1578 & 0.0291 \end{bmatrix}$$

and $\varepsilon_{tj} \sim N(0, \sigma^2{}_j)$

$y_1 = -0.0079\ \beta_{01} + 1.1578\ \beta_{11} + \varepsilon_{t1}$

$y_2 = 0.0007\ \beta_{02} + 0.0291\ \beta_{12} + \varepsilon_{t2}$

TABLE 3.3: Fitted coefficients of MSDR model

β	Estimate	Std. Error	P-Value
β_0 (regime 1)	-0.0079	0.0160	0.62145
β_1 (regime1)	1.1578	0.5763	0.04454
σ^2 (regime 1)	0.1045	0.0901	0.24613
β_0 (regime 2)	0.0007	0.0021	0.7389
β_1 (regime 2)	0.0291	0.0492	0.5542
σ^2 (regime 2)	0.6407	0.0733	<2e-16

(Regime 1): Residual standard eror: 0.1733466, Multiple R-squared: 0.04552

(Regime 2): Residual standard error: 0.01873117, Multiple R-squared: 0.5365

From the fitted coefficients in Table 3.3, the regime-specific model equations are as follows:

Regime 1(low-variance regime)
Change in Inflation = -0.0079 + 1.1578 × change in USD exchange rates + ε_t , $\varepsilon_t \sim N(0, 0.1045)$

Regime 2(high-variance regime)

Change in Inflation = 0.0007 + 0.0291 × change in USD exchange rates + ε_t , $\varepsilon_t \sim N(0, 0.6407)$

Utilizing these regression equations, the prediction of future values of the dependent variable is possible. The state-specific variance indicates that the residual errors of the model are distributed Normally around a zero mean and a variance that switches between two values depending on the presiding state of the underlying Markov process.

The Markov state transition matrix P is:

$$P = \begin{bmatrix} 0.9906 & 0.0094 \\ 0.0238 & 0.9762 \end{bmatrix}$$

From the transition probability results, when the inflation rate is in Regime 2, at time t, given that it was in Regime 1 at the previous time step(t-1), is 0.0094, which indicates a low chance of it happening, while that of moving from Regime 2 to 1 is 0.0238. The loglikelihood estimate of the Markov switching dynamic model is 249.6585, and the AIC is -487.317.

The regression diagnostics to check for the validity of the residual assumptions are shown in Figure 3.6. The residual diagnostics plots of the Markov switching regression model show no autocorrelation as the residuals appear to be white noise and fit the Normal distribution.

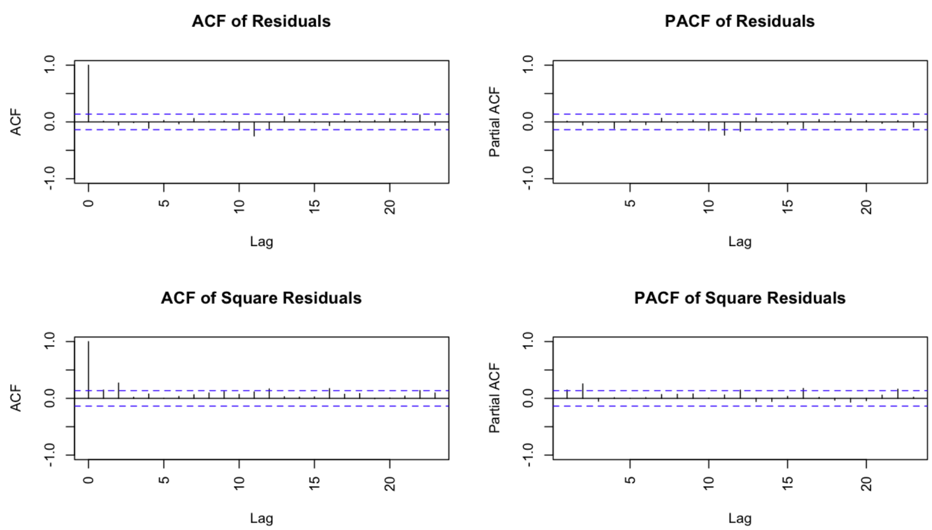

FIGURE 3.6: Plot of residuals MSDR model diagnostics

3.4 Comparative Analysis of the Models

The parameter estimates of the models used in the data analysis, with their standard errors and *p*-values, are presented in Table 3.4. The parameter estimation methods used for the models are; the method of least squares for the OLS regression model and maximum likelihood for the MSDR model.

TABLE 3.4: Parameter estimates of the OLS and MSDR models

Model	β	Estimate	Std.Error	P-value	R^2	σ(RSE)
OLS				0.0549	0.01786	0.1349
	βconst.	-0.003995	0.009504	0.6747		
	βx	0.520168	0.269439	0.0549		
MSM(Reg.1)				0.0445	0.04552	0.17334
	βconst.	-0.0079	0.0160	0.62145		
	βx	1.1578	0.5763	0.04454		
MSM(Reg.2)				0.5542	0.5365	0.01873
	βconst.	0.0007	0.0021	0.7389		
	βx	0.0291	0.0492	0.5542		

4. DISCUSSION

The parameter interpretation of the linear regression shows that for a 1-unit increase in USD exchange rate, the inflation rate increases by 1.1578 in Regime 1. While in Regime 2, a 1-unit increase in USD exchange rate shows an increase in inflation rate by 0.0291. The transition probabilities matrix shows high values at P_{11} and P_{22}, meaning that the probability of remaining in a particular state once it transits to that state is very high.

The significant measures and the goodness-of-fit statistics of the ordinary least squares regression model and the Markov switching dynamic regression model (regimes 1 and 2) show that MSM (regime 2) has the lowest residual standard error among the models. This statistic measures the standard deviation of the residuals in a regression model. The R-squared metric, which measures how effectively the model used for the analysis captures the dependent variable, shows that MSM (regime 2) has the highest value. Modern applications of time series analysis using convolutional neural networks, a non-linear generalization of autoregressive models or application of data stream analysis, a natural blend of practical time series analysis, and machine learning for further comparison.

5. CONCLUSION

The ordinary regression model and the Markov switching dynamic regression model were used to check for a correlation between monthly percentage inflation rates in Nigeria and the monthly USD exchange rates, and the p-values of both models indicate that the independent variable is statistically significant at a 95% confidence level. The variance-switching MSDR model was used to check for variability in the observed values of the dependent random variable (inflation rates) using the USD exchange rates as the explanatory regression variable. Because Markov switching models incorporate regime switching in the parameter space, the maximum likelihood estimation method is adequate to estimate the optimal values of the parameters across the states.

The transition probabilities of the MSDR model show that when the inflation rate is in a high variance regime, the chances of switching to a low variance regime are 2.38%, and the chances of inflation rates moving from a low to a high variance regime are 0.94% of the time. Also, when the Markov state model is in a particular state, inflation rates persist longer in that regime before switching to another state. The results show that the hidden Markov portion of the MSDR model better explained the variability in the inflation rates using the USD exchange rates than the ordinary regression model. The estimates of the regression variable from the fitted models showing the extent to which exchange rates affect inflation rates give a more realistic value in MSDR than in OLS. Therefore, the Markov switching dynamic regression (regime 2) model, which represents the high-variance regime, explains the regression objectives more than the OLS regression model, as shown in the results. Consequently, it shows that when the Markov state model is in the high variance regime, Nigeria's economy shows a recession. The results show that the regime-switching model outperformed the single regime model in capturing the properties of the data from the comparative analysis.

Acknowledgment.
The authors express their gratitude to the anonymous reviewers for their useful comments and suggestions.

Authors Contributions. Each author contributed equally in the work.

Authors' Conflicts of interest. None

Funding Statement.
There is no funding conflict.

REFERENCES

[1] K. Chung-Ming. Lecture on the Markov switching model. Institute of Economics Academia Sinica. https://www.sinica.edu.tw/as/ssrc/ckuan. (2002).

[2] D. Jurafsky and J.H. Martin. Speech and Language Processing. https://www.stanford.edu. (2021).

[3] B. Mor, S. Garhwal, Ajay Kumar. A systematic review of hidden Markov models and their applications. Archives of Computational Methods in Engineering. 3(2020) 1429-1448.

[4] P. Pomorski and D. Gorse. Improving on the Markov-Switching Regression Model by the Use of an Adaptive Moving Average. Computational Engineering, Finance, and Science. https://doi.org/10.48550/arXiv.2208.11574(2022).

[5] M. T. Kartal, F. Ayhan and D. Kirikkaleli. Regime-switching effect of COVID-19 pandemic on stock market index: evidence from Turkey as an emerging market example. Macroeconomics and Finance in Emerging Market Economies. https://doi.org/10.1080/17520843.2022.2091825(2022).

[6] D. Zivkov, J. Duraskovic and S. Manic. How do oil price changes affect inflation in Central and Eastern European countries? A waveletbased Markov switching approach. Baltic Journal of Economics. 19(1)(2019) 84-104.

[7] S. G. Tesfamichael and Y. A. Shiferaw. A Markov regime- switching regression approach to modelling NDVI from surface temperature and soil moisture. International Journal of Remote Sensing. 40(24)(2019) 9352-9379.

[8] G. Costa and R. H. Kwon. Risk parity portfolio optimization under a Markov regime-switching framework. Quantitative Finance. 19(3)(2019) 453-471.

[9] R. Alvarez, M. Camacho and M. Ruiz. Inference on Filtered and Smoothed Probabilities in Markov-Switching Autoregressive Models. Journal of Business & Economic Statistics. 37(3)(2019).

[10] K. Carstensen, M. Heinrich , M. Reif , M. H. Wolters. Predicting ordinary and severe recessions with a three-state Markov-switching dynamic factor model: An application to the German business cycle. International Journal of Forecasting. 36(3)(2020) 829-850.

[11] V. Moutinho, H. Oliveira, J. Mota. Examining the long term relationships between energy commodities prices and carbon prices on electricity prices using Markov switching regression. Energy Reports. 8 (2022) 589–594.

[12]Y. Kim, L. Huo and T.H. Kim. Dealing with Markov-switching parameters in quantile regression models. Communications in Statistics: Simulation and Computation. 51(11)(2022) 6773 - 6791.

[13] J. Stillwagon and P. Sullivan. Markov switching in exchange rate models: will more regimes help?. Empirical Economics. 59 (2020) 413 - 436.

[14] S.T. Hwu, C. J. Kim, and J. Piger. An N-State Endogenous Markov-Switching Model With Applications In Macroeconomics And Finance. Macroeconomic Dynamics. Cambridge University Press, 25(8)(2021) 1937-1965.

[15] T. Adam, A. Mayr, and T. Kneib. Gradient boosting in Markov-switching generalized additive models for location, scale, and shape. Econometrics and Statistics. 22 (2022) 3-16.

[16] A. Rahman, M. A. Khan, and L.Charfeddine. Financial development-economic growth nexus in Pakistan: new evidence from the Markov switching model. Cogent Economics & Finance. 8(1)(2020).

[17] M. K. Anser, D.I. Godil, M.A. Khan, A.A. Nassani, K. Zaman, and M. M. Q. Abro. The impact of coal combustion, nitrous oxide emissions, and traffic emissions on COVID-19 cases: a Markov-switching approach. Enviromental Science and Pollution Research. 28(2021) 64882–64891.

[18] M.I. Shah, D. Kirikkaleli, F. F. Adedoyin. Regime switching effect of COVID-19 pandemic on renewable electricity generation in Denmark. Renewable Energy. 175 (2021) 797-806.

[19] D. Degras, C. M. Ting, and H. Ombao. Markov-switching state-space models with applications to neuroimaging. Computational Statistics & Data Analysis. 174(2022).

[20] D. Sachin. A Math Lover's Guide to Hidden Markov Models. https://www. towards datascience.com (2021a).

[21] D. Sachin. A worm's eye-view of the Markov switching dynamic regression model. https://www.towards datascience.com (2021b).

[22] A. Ibn. Introduction to Markov Regime Switching Model. https://www.rpubs.com (2016).

ROSEMARY UKAMAKA OKAFOR*
DEPARTMENT OF STATISTICS, UNIVERSITY OF LAGOS, AKOKA, LAGOS STATE, NIGERIA.
E-*mail address*: ukagbans@gmail.com

JOSEPHINE NNEAMAKA ONYEKA-UBAKA
DEPARTMENT OF STATISTICS, UNIVERSITY OF LAGOS, AKOKA, LAGOS STATE, NIGERIA.

E-*mail address*: jonyeka-ubaka@unilag.edu.ng
.