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OVERVIEW OF SCHEDULING PROBLEMS WITH LEARNING EFFECT, DETERIORATING EFFECT, MAINTENANCE ACTIVITY AND NON-MONOTONIC TIME-DEPENDENT PROCESSING TIMES

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ABSTRACT. In recent times many research has been focused on assumption that processing times of a job is unfixed. In practice, thus the processing time of a job processing time of a job may changes due to some factors. We investigated factors that affect the varying processing time of a job under different machine environments. We were able to identify four factors that may influence processing time of a job, these include learning effect, deteriorating effect, maintenance activity and non-monotonic time-dependent processing time. In this paper, we discussed machine environments and their assumption and some features of jobs processing times. We also gave a concise overview on the literature on scheduling with learning effect, deteriorating effect, maintenance activity and non-monotonous time dependent processing times. The emphasis was on single machine, parallel machine and flowshop machine while few work are done on open shop and job shop machines. We discovered that these factors are never been considered simultaneously. Dynamic programming and machine learning approach methods are not been used while hybrid meta-heuristics methods are scarcely used in all the machine environments under these constraints.

1. INTRODUCTION

In a competitive market, when there is a high demand for a particular product, time to meet with customer's demand for such a product is essential. In order to meet the due windows given to the customers in making the products available, it is imperative to schedule the machines. This schedule is aimed at achieving high utilization of available resources to minimize customer's wait time for a particular product and meeting delivery dates. The application of scheduling is useful in the area of production, manufacturing, transportation, logistics, service and telecommunication.

In recent times, researchers are working earnestly to solve the complexity of the problems associated with scheduling processes. These involve setting an objective function subject to constraints given that n jobs $(J_1, J_2, J_3, \dots, J_n)$ is scheduled on m machines either weighted or unweighted (see Adamu & Adewunmi (2016)). As cited by (Priore et al., 2014), scheduling problems include three components: a machine environment, specific job characteristics, and one or more optimality criteria (Brucker, 2001). Graham et al. (1979) classified scheduling problems into three-field

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notation $\alpha|\beta|\gamma$, where α describes the machine environment, β describes the job characteristics, and γ describes the objective criterion to be minimized. The machine environments can be classified into a single machine and multiple machine scheduling environments. The multiple machine environment is broadly classified into parallel machines, flowshop machine, job shop and open shop.

The classical assumption that job processing times are known and fixed during the period of job processing in scheduling models may not be realistic in some cases, since processing times of jobs changes due to learning effect (Biskup, 2008), deteriorating/ageing effect (Janiak et al., 2011) and resource allocation (Shabtay & Steiner, 2007; Janiak et al., 2007). The processing times of jobs reduced as a result of processor's learning over time while the processing times of jobs lengthened due to deteriorating effect of the machines (Sun, 2009). This phenomenon is described as the time-dependent processing time (Yin et al., 2014).

Biskup (1999) proposed the notion of learning into the field of scheduling which confirmed that single-machine scheduling with a learning effect can be solved as a polynomial if its objective is to minimize the variation of job finish times from a common due date or to minimize the sum of job flow times. The author assumed a learning process that is exhibited in a decrease in production time as a function of the number of repetitions of the production of a single item. The learning effect in scheduling may have greater effects on firms which produces similar jobs on one machine or identical parallel machines for some customers. In many cases, variation of normal job processing times may be due to differing quantities or somewhat different components that constitutes the products. Nevertheless, adoption of batch processing of job helps to build capacity of the workers to improve on job tasks, machine operations and efficient management of software needed in the processing of raw materials, hence speed is gained in the process and larger job scope can be managed (Wang and Xia, 2005).

Attentions are been drawn recently to the machine scheduling with deterioration processing times. Browne and Yechiali (1990) introduced a scheduling problem with the constraint of deteriorating jobs where the processing time of a job is considered to be a linear non-decreasing start-time dependent function. The delay in processing of a job attracts penalty that may cause elongation of the completion of the task; this can be applicable in areas of scheduling maintenance jobs, cleaning assignments and steel production (Huang et. al., 2012). In practical sense, the processing time of a job is non-monotonic, caused by changes in environment conditions. An example is vehicle delivery times (Azadeh et al., 2017). In the manufacturing system, scheduling of a job can be done by fixed or variable maintenance. For the fixed maintenance the starting time of a job and the maintenance duration are fixed and known in advance, however, the starting time is set by the scheduler, and the maintenance duration is a non-decreasing function its starting time in the variable maintenance (Ma et al. 2010).

1.1. **Machine Environments.** There are two main machine environments namely – single machine and multiple machines. The multiple machines can be broadly categorized into parallel machines, flowshop machines, open shop machines and job shop machines.

1.1.1. **Single Machine Scheduling ($\alpha = 1$).** A single machine scheduling is the process of designating a group of tasks to a single machine or resource. The tasks are organized in such a way that one or many performance measures may be optimized and the performance measures are earliness, lateness, flow time, tardiness etc. The jobs in the single-machine are described by the processing time (p_j), release date (r_j) – the time at which job j is available for processing, and the due date (d_j) –time at which the processing of job j is due to be completed. Among the problems associated with a single machine, problems related to earliness and tardiness are important in production planning because earliness causes inventory and maintenance carrying costs while tardiness causes to customer's dissatisfaction and loss of customers' goodwill (Mohamad and Said, 2011). Examples of single machines are printing press for printing and binding, photocopying facility, a tailor who receives orders for stitching garments.

The following are the assumptions of a single machine scheduling:

1. Machine is continuously available throughout the scheduling period.
2. The machine processes jobs one at a time
3. The process time of each job on the machine is precisely known and it is independent of preceding jobs.
4. The process time includes set up time and exact processing time.
5. The other job related information is previously known. This information includes the release date (r_j) and the due date of the job (d_j).
6. A non-preemptive scheduling i.e. interruption is not permitted during jobs processing.

1.1.2. **Multiple Machine Scheduling.** In scheduling theory, there are three basic types of multiple machine scheduling models - parallel systems, serial (flow shop) systems, and hybrid (job shop and open shop) systems. We shall briefly discuss each of these multiple machines scheduling models.

1.1.2.1. **Parallel Machine Scheduling.** A scheduling problem for parallel machines is basically made up of two steps - allocation of jobs to machines and generating a sequence of the jobs on a machine. The parallel machines can be classified into three categories based on their operations –identical machine ($\alpha = P_m$), unrelated machine ($\alpha = R_m$), and uniform parallel machines ($\alpha = Q_m$). The basic model for the parallel machine scheduling includes n jobs are simultaneously available at time zero, m parallel machines available for processing, a job can be processed by at most one machine at a time. In identical parallel machines, a job can be processed at any one of the m machines with different processing times (Nandagopal. et al., 2016). The independent parallel machines sequencing require no distinct association between the processing times in the different machines. This implies that no proportionality between the processing time of a task on a given machine and the time of the same task on another machine. That is, $p_{ij} = \frac{p_i}{s_{ij}}$ where p_{ij} is the processing time of job j on the machine i and s_{ij} is the speed of job j on the machine.

Santos et al. (2012). In uniform parallel machines, there are m parallel machines with different speeds $s_1,\ s_2, s_3, \ldots, s_m$ such that $s_1 < s_2 <\ s_3 < \ldots <\ s_m$. The processing time (p_{ij}) of the job j on the machine 1 is given as $p_{ij} = \frac{p_{1j}}{s_i}$ $\forall$ i and j. The completion time for a given job is inversely proportional to the speed of the machines $\frac{1}{s_1}:\frac{1}{s_2}:\frac{1}{s_3}:\ldots:\frac{1}{s_m}$.

Santos et al. (2012) highlighted some assumptions in the scheduling problems in parallel machines as follows:

1. A job is completed when processed by any of the machines
2. A machine can process only one job at a time.
3. Job interruption is not allowed, so each job once started must be completed.
4. All jobs are available from time zero.
5. Accumulation of job is allowed – job can wait for a free machine.
6. Processing time is independent of the sequence
7. Downtime of machines is allowed.
8. The number of jobs is known and fixed.
9. The number of machines is known and fixed.
10. The processing times are known and fixed.
11. All specifications to define a particular problem are known and fixed.

1.1.2.2. **Flowshop Machine Scheduling ($\alpha = F_m$).** In a flow shop problem, there are *m* machines that should process *n* jobs. All jobs have the same processing sequences through the machines. The order of the jobs on each machine can be different. An example of a flowshop is assembly lines. However, when each job to be processed has its routing it is called *Job Shop* and when jobs have no specific routing it is known as *Open Shop*.

Baker and Trietsch (2009) stated the following conditions for flowshop machines:

1. A set of *n* unrelated, multiple-operation jobs are available for processing at time zero. (Each job requires *m* operations, and each operation requires a different machine.)
2. Setup times for the operations are independent of sequence and included in processing times.
3. Job descriptors are known in advance.
4. All machines are continuously available.
5. Once an operation begins, it proceeds without interruption.

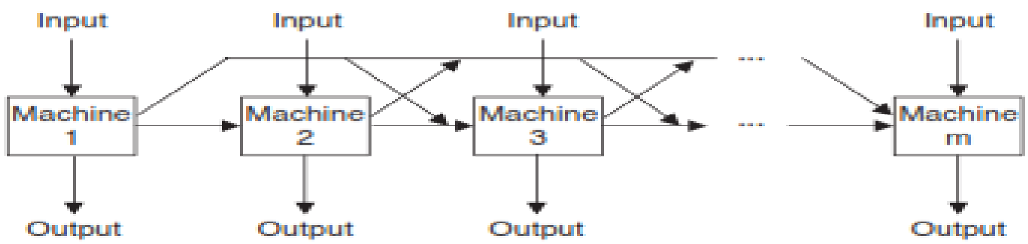

FIGURE 1. Workflow of tasks in a general flowshop machines environment Baker and Trietsch (2009)

The remaining sections of this article are arranged as follows: Section 2 explains some of the features of job processing times and an overview of available literature.

Section 3 contains the concluding statements thus revealing the gaps for future exploration.

2. LITERATURE REVIEW

Processing time of a job is determined by some features such as the learning effect of the processor, the deteriorating effect of the machine, the non-monotonic time-dependent processing times of a job and the maintenance activity. In this subsection, each of these features shall be discussed.

2.1. **Learning effect.** The phenomenon of learning effect where was first introduced by Wright (1936) that demonstrated the improvement in the efficiency of a processor. The concept was brought to scheduling problems by Biskup (1999) and the author proposed that the time of production is a function of the number of repetitions of the production of a single item with a decreasing learning effect (Biskup, 1999). Learning effect determines the processing time of a job, the processor gain more knowledge as a result of repetition of similarly tasks. The more the experience gained; the shorter the processing time of the next jobs.

Learning effects could be originated from the experience gained by the processor during the production process or from the time taken for the machine to completion a task. The former scenario is referred to as sum-of-processing-time approach while the latter is the position-based learning. Learning effects could be as a result of change in production – inexperience workers, purchase of new machines or replacement of equipment by technically refined machines, change in workflow, and processing of new jobs.

In addition, Biskup (1999) developed a model that the processing time of a job depends on position of the job in a sequence and it can be mathematically written as:

$$p_{j[r]} = p_j r^{-\alpha} \tag{1}$$

where $p_{j[r]}$ denotes an actual processing time of job j sequenced in rth position, p_j is a normal processing time (without learning) of job j and $\alpha > 0$ is a learning index.

2.1.1**. Sum-of-Processing-Time-Based Learning Effect.** This is one of the approaches to learning used in scheduling, it takes into cognizance, the prior knowledge gained from the workers to process the jobs. Wang, Wang, and Zhang (2010) investigated a single machine scheduling problem with sum-of-processing-time-based learning effect and heuristic algorithms to minimize the weighted sum of completion time and the maximum lateness. Koulamas and Kyparisis (2007) studied a sum-of-processing-time-based learning model for single-machine and two-machine flowshop scheduling problems. The authors proved that both problems can be solved in polynomial time and that the shortest processing time sequence is the optimal schedule for the objective of minimizing both the makespan and the total completion time.

2.1.2. **Position-Based Learning Effect.** This approach assumes that the processing time of the job is independent of the operators but rather driven by the machine with no human involvement, e.g. processing of memory chips. Zhang, Wu, and Zhou (2013) studied a single-machine stochastic scheduling problems with position-based learning effect and investigated the optimal permutation policies for the problems with and without breakdowns.

2.1.3**. Time-Dependent Learning Effect.** When the processing time of a job is flexible and also dependent on the starting time of the job, then it is referred to as time-dependent scheduling. In time-dependent learning effect, the processing time of a job is determined by a function of its corresponding starting time and positional sequence in each machine. Invariably, the processing time of a job depends on both its starting time and the total normal processing time of the prior processed jobs. This can be represented as:

$$p_{j[r]} = p_j(1 + p_{[1]} + p_{[2]} + \cdots + p_{[r-1]})^{\alpha}, \quad (2)$$

where $\alpha \leq 0$ is a constant learning index.

For example, the production of porcelain craftworks, the clay and coagulant are used to shape the designs, this raw materials become harder within an interval of time, and then the processing time increases (Wang, et al., 2012). Kuo and Yang (2006a) investigated a time-dependent learning effect on a single-machine scheduling problem to minimize the total completion time. They assumed that the time-dependent learning effect of a job is a function of the total normal processing time of the jobs scheduled in front of it in the sequence. They showed that short processing time sequence is optimal with the objective function. Kuo and Yang (2006b) also considered a time-dependent learning effect on a single-machine scheduling problem to minimize the makespan and total completion time, and they proved that the problem remains polynomially solvable.

2.1.4. **Job-Dependent Learning Effect.** In the job-dependent learning effect, it is assumed that each job has its unique learning index, thus the learning process of a processor may be greatly affected by the kind of job to be processed. Mosheiov and Sidney (2003) modified Biskup (1999) model and assumed that each job has its own learning index α_j and it can be written as:

$$p_{j[r]} = p_j r^{-\alpha_j} \quad (3)$$

where $p_{j[r]}$ denotes an actual processing time of job *j* sequenced in *r*th position, p_j is a normal processing time (without learning) of job j and $\alpha_j > 0$ is a learning index of individual job.

Janiak and Rudek (2005) studied existing learning models and introduced experience of the processor to the models to come up with the generalization of Mosheiov and Sidney (2003) model. This model can be expressed as:

$$p_{j[r]} = p_j\left(\sum_{l=1}^{r} e_{[l]}\right)^{-\alpha_j} \quad (4)$$

where $\sum_{l=1}^{r} e_{[l]}$ is an experience of a processor at the moment when processing of the job scheduled in *r*th position is started (this experience is gained by processing the jobs from positions 1,...,r).

Some recent literature on the learning effect under single machine are: Lee, Wang and Su (2015), Hosseini and Tavakkoli-Moghaddam (2013), Wang,Wang and Zhang (2010), Li and Hsu (2012), Wang (2008), Mosheiov (2001a) and Biskup (1999). Biskup (1999) considered a job-position based learning model for the processing time of a job on a single machine as a decreasing function of its position in the sequence. His objective was to minimize the deviation from a common due date or minimize the total flow time, and he showed that the single machine scheduling problem is polynomial time solvable. Subsequently, Mosheiov (2001) built upon Biskup's model, he provided polynomial solutions to the single-machine scheduling problem by minimizing the makespan.

Lee, Wu and Sung (2004) proposed model for a bi-criteria single machine problems is polynomial time solvable for the objective of minimizing the sum of the total completion time and the maximum tardiness. Wang and Cheng (2000) studied a single machine scheduling problem with a volume-dependent piecewise linear processing time function to model the learning effects. They demonstrated that the maximum lateness problem is NP-hard and polynomial time solvable. They proposed two heuristic algorithms and provided their worst case performances. Wu and Lee (2007) proposed a model to investigate two single- machine scheduling problems to minimizing makespan and the total completion time. They showed that the objective functions of makespan and the total completion time are both polynomially solvable. They also showed that the total weighted completion time is polynomially solvable under certain agreeable conditions.

Xingong and Guangle (2010) proposed a general learning model for the single-machine scheduling problems with position-based learning and the sum-of-processing-time-based learning effects simultaneous. They provided the polynomial time optimal solutions for the problems of minimizing the makespan and the total completion time. They also showed that the problems of minimizing the total weighted completion time and the total tardiness are polynomially solvable under certain agreeable conditions. Liu *et al.* (2015) investigated a single-machine problem with sum-of-processing-times-based learning and ready times to minimize the makespan. They built a branch-and-bound algorithm to find the optimal solution and a heuristic algorithm to find the near-optimal solution. Also, Koulamas and Kyparisis (2007) proposed a sum-of-processing-time-based learning model on a single machine scheduling. They showed the problem is polynomially solvable if objectives are makespan and the total completion time. Lee, Wu and Hsu (2010) investigated a single machine position-based learning scheduling problem with ready times to minimize the total makespan and total completion time.

Wang, Wang and Wang (2021) worked on a single-machine resource allocation scheduling problem with learning effect and group technology. They considered a slack due-date assignment with objective of determining the optimal sequence of jobs and groups, optimal due-date assignment, and optimal resource allocation such that the weighted sum of earliness and tardiness penalties, common flow allowances, and resource consumption cost is minimized. They proposed heuristic, tabu-search and branch-and-bound algorithms and showed also that this problem can be solve in polynomial time.

On the parallel machine environment, Mosheiov (2001) explained the learning effect in the context of multiple machines. He showed that the problem of minimizing the makespan with learning effect on parallel identical machine is NP-hard. Although, he claimed that the complexity of minimizing flow-time with a learning effect on parallel identical machines seems less trivial. He showed that the short processing time rule does not remain optimal with learning effect; this is contrary to the classical version. Mosheiov and Sidney (2003) proposed a learning model to minimize the total flow-time in an unrelated parallel machine scheduling problem, they showed that the problem is polynomially solvable.

Eren (2009) proposed heuristic and mathematical programming model to minimize the maximum lateness with learning effect. Wang and Hsu (2016) considered scheduling problems with general truncated job-dependent learning effect on unrelated parallel-machine to minimize total machine load, total completion (waiting) time, and total absolute differences in completion (waiting) times. They found that if the number of machines is fixed, the problems can be polynomially solved with $O(n^{m+2})$, where n is the number of jobs and m is number of machines.

Lee, et al. (2012) studied uniform parallel machine problems to minimize makespan by jointly find an optimal assignment of operators to machines and an optimal schedule. They proposed two heuristic algorithms to evaluate their performances and they found that Particle Swam Optimization outperformed Genetic Algorithm. Przybylski (2018) introduced a model of parallel-machine scheduling with job processing times described by proper Riemann integrals of a given function. They formulated and proved few properties of that model and showed that some problems of parallel-machine scheduling of jobs with integral-based learning effect can be solved using polynomial algorithms applied earlier to fixed job processing times.

Lin and Chuang (2015) investigated the problem of scheduling jobs on uniform parallel machines to minimize total weighted tardiness under a truncation sum-of-logarithm-processing-times-based learning effect. In their proposed learning model, the actual job processing time is a function that depends not only on the processing times of the jobs already processed but also on a control parameter. They proposed iterated local search and compared result with other metaheuristics (simulated annealing and ant colony optimization). They showed that the proposed iterated local search outperformed other metaheuristics under consideration.

Besides, Wang and Xia (2005) considered some flow-shop scheduling problems with learning effect to minimize one of the makespan and total flowtime. A heuristic algorithm with worst-case bound m for each criteria is known, where m is the number of machines. They proposed a polynomial algorithm for identical processing time on each machine and an increasing series of dominating machines. They showed that the classical Johnson's rule is not the optimal solution for the two-machine flowshop scheduling to minimize makespan with a learning effect. Lai *et al.* (2014) investigated two-machine flowshop problem with a truncated learning effect to find an optimal schedule to minimize the total completion time. They incorporated a branch and-bound algorithm with a dominance property and developed four lower bounds to derive the optimal solution. They showed that the branch-and-bound algorithm can solve instances up to 18 jobs, and the proposed simulated annealing algorithm performs well in item of CPU time and error percentage.

Tian *et al.* (2018) studied the no-wait permutation scheduling problem with the learning effect and resource allocation. They provided a bi-criteria analysis for the two-machine scheduling, to minimize the schedule criterion and to minimize the total weighted resource cost. They showed that the two problems are polynomially solvable. Chen *et al.* (2006) considered bi-criteria two-machine flowshop scheduling problem with learning effect to minimize a weighted sum of the total competion time and the maximum tardiness. They used branch-and-bound method with dominance properties and a lower bound to search for the exact solution for small job-size problems. Two heuristic algorithms are proposed to overcome the inefficiency of the branch-and-bound algorithm for large job-size problems. The proposed branch-and-bound algorithm performed well up to job size 18 and the proposed heuristics are accurate in solving the problem.

Bai, Tang, Zhang and Santibanez-Gonzalez (2018) proposed a mixed integer programming (MIP) model and branch-and-bound algorithm for the learning effect flowshop scheduling problem with release date to minimize makespan, total completion time and total quadratic completion time. They demonstrated that for large-scale problem, the asymptotic optimality of a class of shortest processing time available (SPTA)-based heuristics is established in terms of probability limit.

2.2. Deteriorating (Aging) Effect. An aging effect is an occurrence of a decline in the efficiency of the processor due to fatigue caused by deteriorating jobs. Gupta and Gupta

(1988) originally introduced the concept of deterioration in scheduling. In classical scheduling, it is assumed that the job processing times are fixed and known through the entire process, but in many production environments, this assumption may not hold as more time is often expended on job processed later than the same job when processed earlier (Lee & Lu, 2012). Therefore, the processing time of a job may be increased as a result of starting time of a job, position of a job or sum of normal processing times of jobs already processed (Janiak et al., 2011). A facility is said to be deteriorating if processing capacity decreases over time and then subsequent jobs processed require an extended processing time than jobs processed earlier (Cheng et al., 2006).

2.2.1. **Linear Deteriorating Effect.** The processing time of a job is restricted to be a monotone increasing linear function that satisfies $p_j(t) = p_j + a_j t$, where p_j is the normal processing time of job J and a_j is the deteriorating rate. This case is known as the linear deteriorating model.
The mathematical model with the job processing time as a linear function dependent on the position of a job in a sequence (Bachman & Janiak, 2004) is given by:

$$p_{j[r]} = p_j + a_j r \quad (5)$$

where $p_{j[r]}$ denotes actual processing time of the job j scheduled in position r, p_j is the normal processing time of the job and $a_j \geq 0$ is an aging ratio of job j .

2.2.2**. Non-linear Deteriorating Effect.** This is similar to the linear deterioration effect, but the actual processing time is a non-linear function that satisfies:

$$p_j(t) = p_j + a_j t^b \quad (6)$$

where $b > 0$ is non-linear deteriorating effect, and it is the measure of increase in the processing time of a job per unit delay in its starting time.
2.2.3 Piecewise Linear Deteriorating Effect
A piecewise function (Y) behaves differently based on different values of input X. In this case, the actual processing time of each job is as a piecewise linear function and the deterioration happens in a period of time after the starting process leading to an increase in the actual processing time. This increment will not continue to the end, but after a specific time, the value of deterioration will be constant until the end of the process (Jafari and Lotfi, 2018). Therefore, the actual processing time of each job is a function of two or more than two constant or linear criteria. This can be represented in equation and chart below.

$$p_i = \begin{cases} a_i\,, & if\ S \leq y_1 \\ a_i + b_i(S - y_1), & if\ y_1 < S < y_2 \\ a_i + b_i(y_2 - y_1), & if\ S \geq y_2 \end{cases} \quad (7)$$

where a_i is the normal processing time and b_i is the deteriorating rate.

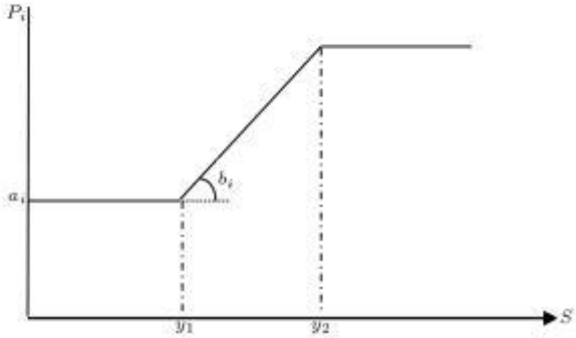

FIGURE 2. Description of Jafari and Lotfi (2018)

Few recent literature found on the single machine scheduling with deteriorating effect only include, Browne and Yechiali (1990),Wu *et al.* (2008), Wang *et al.* (2008a), Wang et al. (2008b), Wang et al. (2009), Wang et al. (2011), and Wang and Wang (2012),Yin *et al.* (2015c) and Salama and Srinivas (2021). However, some researchers considered the combination of learning and deteriorating effects on the single machine scheduling are: Wang et al. (2009), Ghodratnama *et al.* (2010), Wang et al. (2010) and Huang *et al.* (2011).

Besides, many authors jointly combined any other two of the aforementioned features of the processing time of a job under a single machine scheduling (see Ying *et al.* (2017), Yang and Yang (2010) and Zhao and Tang (2010)). Azadeh *et al*. (2017) studied the joint combined the learning effect, deteriorating effect and non-monotonic processing times on a single machine with the objective of minimizing the total tardiness of jobs. In addition, the similar pattern observed in the literature based on the parallel machine scheduling. Ji, *et al.* (2016) considered parallel machine scheduling with deteriorating jobs and DeJong's learning effect. They provided a fully polynomial time approximation scheme to minimize the makespan and they showed that the total completion time minimization problem is polynomially solvable. Other literature that on a single machine scheduling problems that focused on both learning and deteriorating effects are: Yan *et al.* (2008), Wang (2009), Huang, *et al.* (2010), Wang and Guo (2010), Wang and Wang (2011) and Wu *et al.* (2011).

Ruiz-Torres et al. (2013) studied unrelated parallel machine scheduling problem to minimize makespan when the deteriorating effect depends on the sequence of the jobs in the machines. Lalla-Ruiz and Voß (2016) proposed two models based on the Set Partitioning Problem (SPP) in identical parallel machines where each job has a deteriorating date and observes a step function for the processing time to minimize the total completion time. Jeng and Lin (2007), Kang and Ng (2007), Ren and Kang (2007), Ji and Cheng (2008), Ji and Cheng (2009), Mazdeh *et al.* (2010), Huang and Wang (2011), Wang and Wei (2011), Liu, et al. (2011), Ruiz-Torres *et al.* (2013), Yin, et al. (2017) investigated deteriorating effect only on the parallel machine scheduling problems.

Besides, Toksari and Guner (2010) investigated the common due-date early/tardy scheduling problem on a parallel machine under the effects of time-dependent learning and linear and nonlinear deterioration. Arik and Toksari (2018) studied a multi-objective parallel machine scheduling problem under fully fuzzy environment with fuzzy job deterioration effect, fuzzy learning effect and fuzzy processing times to minimize total tardiness penalty cost, earliness penalty cost and cost of setting due dates.

Wang *et al.* (2008) considered the flow shop scheduling problems with the learning and deterioration effects to minimize makespan and total completion time. They established that some special flowshop scheduling problems are polynomially solvable. Wang and Liu (2009) proposed a two-machine flowshop scheduling problem with learning effect and deterioration effect to minimize total completion time. They used a heuristic algorithm and showed that computational experiments performed effectively and efficiently in obtaining near-optimal solutions. Other literature that considered both learning and deteriorating effects are: Wang (2006), Wang *et al.* (2012), Low and Lin (2013) and Lu (2016).

2.3. **Non-monotonic Processing Time.** A non-monotonic function is a function that is increasing and decreasing on different intervals of its domain. An example of a non-monotonic function is $f(x) = x^2$. The processing time of a job is said to be a non-monotonic if it cannot be defined as a monotonic function of its start time. This phenomena may be caused as a result of instability in environmental conditions such as temperature, weather condition, and illumination that affect operator's performance (Azadeh et al. , 2016). This is referred to as a *breaking point*.

Farahani and Hosseini (2013) studied the combination of learning effect, deteriorating effect and non-monotonic processing time. The actual processing time of job j is calculated as follows:

$$p_j^A = \begin{cases} p_j^{ld} + (a - c_{j-1})\beta & if\ c_{j-1} \leq a \\ p_j^{ld} + (c_{j-1} - a)\beta\ , & otherwise \end{cases} \tag{8}$$

where p_j^{ld} represents the processing time of job j allowing for learning effect and deterioration,

a represents the breaking point,

$0 \leq \beta \leq 1$ represents the job-independent increasing rate of the processing time

c_j represents the completion time of job j

Low, C. and Lin, W-Y. (2013) proposed a model that the processing time of a job is determined by a function of its corresponding starting time and positional sequence in each machine. They showed that some single machines and flowshop scheduling problems are polynomially solvable with performance measures of makespan, total completion time, and weighted completion time. Farahani and Hosseini (2013) investigated a single machine scheduling problems, considering non-monotonic time-dependent processing times of a job to minimize the cycle time using heuristic method. Also, Azadeh *et al.* (2017) proposed a single machine with learning effect, deteriorating effect and non-monotonic processing times to minimize total tardiness using hybrid of GA-TS methods.

2.4**. Maintenance Activities.** Some jobs necessitate to be processed by specific tool but due to the ageing effect the tool needs to be restored to its original condition through maintenance or replacement. The normal processing time of a job is taken if the job is scheduled immediately after maintenance (replacement). Then, the due to the aging effect hen may cause the processing time of the subsequent jobs to be prolonged (Kuo and Yang, 2008).

The exact maintenance duration is shorter if the maintenance is arranged later in the sequence due to the learning effect (Yang and Yang, 2010). The actual maintenance time in the position-dependent learning effect when the maintenance is the *ith* maintenance in the sequence is given as:

$$m_i = t_0 i^b \text{ for } i = 1, 2, \dots, k \qquad (9)$$

where t_0 is the maintenance time and $b > 0$ is the learning effect.

Many scheduling literatures assumed that the machines are continuously available over the scheduling horizon while this assumption may be void in practice. In real production, machines may be available out of preventive maintenance, periodic repairs, or breakdowns (Yang, 2013). His study came from the point of view of metal or wood process that cuts products to various sizes and shapes in a parallel-machine environment. The actual processing time is prolonged as the number of products already processed is increased, therefore, the tool is required to be replaced or maintained to improve efficiency of the production process. In a nutshell, maintenance activity is one of the key useful resources in operations management (see Chiu, *et al.* (2004), Joo and Min (2011), where they applied methods to solve maintenance problems in aircrafts.

Yang and Yang (2010) and Zhao and Tang (2010) investigated a single machine scheduling problems to minimize the makespan with deteriorating and maintenance activity, and they showed that the problems can be solved in polynomial time. Yang *et al.* (2012) and Zarook and Abedi (2014) considered the parallel machine scheduling problems under the effect of deterioration and maintenance activities to show that the problems are polynomially solvables. Nouri *et al.* (2013) worked on the flowshop machine environment with learning effect and maintenance activity to minimize the sum of tardiness and maintenance costs using heuristic and hybrid meta-heuristic algorithms. In addition, Najari *et al.* (2016) investigated a flowshop machine scheduling problems in the combination of learning effect, deteriorating effect and maintenance activity to minimize the makespan using both integer linear programming and genetic algorithm.

3. CONCLUSION

This paper gives a holistic overview of scheduling problems under different kind of machine environments- single machine, parallel machine, flowshop machines, job shop and open shop. Many studies have been done on these machine environments with a fixed job processing times. However, there are some real life situations whereby the processing time of jobs is varied under certain features. These features are the time-dependent processing times such as learning effect of the processor, job deteriorating effect, the non-monotonic time-dependent processing times of a job and the maintenance activity. Some scholars studied each of these features separately while some researchers combined more than one feature simultaneously.

In the single machine environment, some of the literatures considered learning effect or /and aging effect while few literature considered non-monotonic processing time or/and maintenance activities. To the best of our knowledge, no literature considered the four features under investigation simultaneously. Also, despite the methods of measuring performances differs, none of them uses dynamic programming method or machine learning approach.

In the case of parallel machine environment, they are limited number of literatures on the non-monotonic processing time and or/and maintenance activities. Most of the literatures on parallel machine are concentrated on the learning effect or/and deteriorating effect, but literatures that focused on more than two of the features under investigation are limited. Thus, a good area for future research.

There are limited research papers on flowshop machine environment with non-monotonic processing time and maintenance activity; this is similar to the case of parallel machine environment. A good number of scholars studied learning effect or/and deteriorating effect under flowshop machine. To the best of our knowledge none of these scholars considered all the four features simultaneously or using any of the machine learning approach to minimize the objective function. This is a green pasture for future research. However, no or limited literature are available on job shop and open shop machine environments and this is also another gap to be filled for future research on the features of job processing times under review.

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